LRNORA

LEARN · REVISE · EXCEL

UNIT I

FUNDAMENTALS OF PHOTONICS

Engineering Physics · Semester I

Savitribai Phule Pune University (SPPU)

BSC-102-BES · 2024 Pattern · First Year Engineering

Complete Unit I · Theory · Derivations · Original diagrams and infographics
Solved numericals · Formula sheet · University question trends · One-page revision sheet

Prepared as original study material by LRNORA, aligned to the official SPPU syllabus (Unit I).
For classroom and self-study use.

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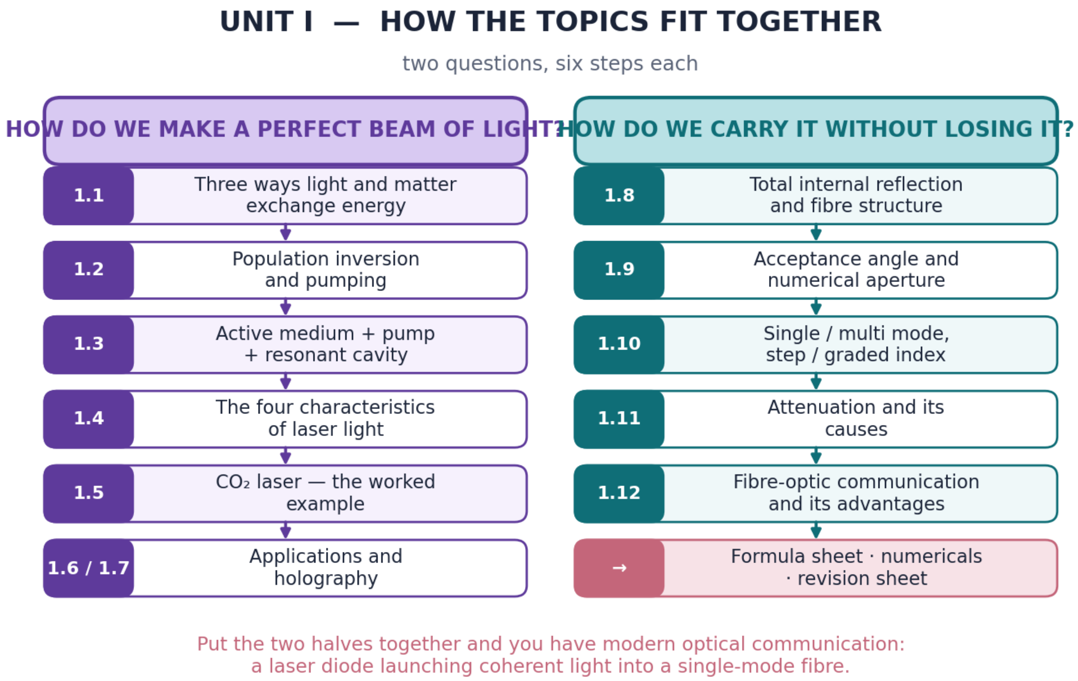
The Official SPPU Syllabus — Unit I

This book covers **Unit I — Fundamentals of Photonics** of Engineering Physics (BSC-102-BES), Semester I of the SPPU 2024 pattern, in full. Nothing prescribed for this unit has been left out, and nothing outside it has been added. Use the table below as a checklist — tick a row only when you can explain that topic without looking at your notes.



How the twelve topics of Unit I fit together.

#	Prescribed topic	Where it is covered
1	Laser: spontaneous and stimulated emission	Topic 1.1
2	Population inversion, pumping	Topic 1.2
3	Active medium and active centre, resonant cavity	Topic 1.3
4	Characteristics of lasers	Topic 1.4
5	CO_2 laser: construction and working	Topic 1.5
6	Engineering applications of laser (IT, medical, industry)	Topic 1.6
7	Holography — recording, reconstruction, applications	Topic 1.7
8	Optical fibres: critical angle, total internal reflection and propagation of laser	Topic 1.8
9	Acceptance angle, acceptance cone, numerical aperture	Topic 1.9
10	Classification: single mode and multimode, step index and graded index	Topic 1.10
11	Attenuation: attenuation coefficient, causes of attenuation	Topic 1.11
12	Advantages of optical fibre communication	Topic 1.12

#	Prescribed topic	Where it is covered
13	Numerical problems on parameters of optical fibre	Numerical Problems section

Unit I — Fundamentals of Photonics · 08 teaching hours.

Course Outcomes (COs)

The syllabus lists five course outcomes for Engineering Physics, one for each unit. **Unit I delivers CO1**; the remaining four are shown so that you can see where this unit sits in the course.

CO	After the course, the learner will be able to...	Unit
CO1	**Develop the understanding of the working principle of lasers and optical fibres and extend it to holography and fibre-optic communication.**	**I ← this book**
CO2	Deduce Schrödinger's wave equations and apply them to problems on bound states by summarising the fundamentals of quantum physics.	II
CO3	Explain the phenomena of interference in thin films, polarisation and double refraction, and connect them to the anti-reflection coating and the LCD.	III
CO4	Develop understanding of the Fermi level and Fermi energy in semiconductors on the basis of Fermi–Dirac statistics and relate them to semiconductor devices; extend the understanding of ultrasonics to thickness measurement and flaw detection.	IV
CO5	Explain the properties of nanoparticles and estimate their engineering applications; explain superconductivity and estimate its engineering applications.	V

Prerequisites assumed by the syllabus

- Bohr's atomic theory and the idea of discrete energy levels.
- Properties of mechanical and electromagnetic waves.
- Huygens' principle and the concept of a wavefront.
- Interference and polarisation of light.
- Wave–particle duality.
- Basic trigonometry, logarithms and calculus.

The examination scheme

Component	Marks	What it covers
Comprehensive Continuous Evaluation (CCE)	30	Class tests, assignments and continuous assessment through the semester.
End-Semester Examination	70	Written theory paper covering all five units; Unit I typically carries about 14 of these marks.
Term Work (practical)	25	Laboratory journal — any 8 experiments from the prescribed list.
Teaching scheme	3 + 2 h/week	3 credits for theory, 1 credit for the laboratory.

Laboratory experiments linked to this unit

- An experiment based on a laser — determining beam divergence, the diameter of a thin wire, or the beam profile.
- An experiment based on an optical fibre — determining the numerical aperture, the attenuation coefficient or the critical angle.

How to use this book

Every topic in this book is built the same way — definition, basic concept, detailed explanation, theory, derivation, diagram, applications, key points, common mistakes, exam tips — so once you are comfortable with the first topic you will know exactly where to find things in all the others. The colour of a box tells you what kind of information is inside it.

	Colour	Box	What it means
◆	Blue	Definition	A formal definition — learn these word-perfect.
∑	Yellow	Formula	An equation, with every symbol and SI unit explained.
●	Purple	Important note	An assumption, a subtlety, or a point students often miss.
✔	Teal	Remember these points	The quick summary at the end of each topic.
⚠	Red	Common mistakes	Errors students make in exams — read before the paper.
★	Green	Exam tips	What SPPU examiners actually look for.
⚙	Grey	Applications / numericals	Real-life uses, and worked problems solved step by step.
↻	Pink	Quick revision box	A compressed recap after each group of topics.

A suggested study order

1. Read the **Learning Objectives** at the start of the unit so you know what you are aiming for.
2. Work through each topic in order — definition first, then concept, then the detailed theory and derivation.
3. Copy every diagram out by hand once. Diagrams carry marks in the SPPU end-semester paper and they are the fastest thing to lose.
4. Do the numericals with the book closed, then check your steps.
5. Two days before the paper, read only the **Formula Sheet**, the **One-Page Revision Sheet** and the **Frequently Asked Questions**.

● **Please read this**

About the question trends. The “Frequently Asked University Questions” section describes the *types* of question that have commonly appeared in SPPU Engineering Physics papers of recent years. They are written fresh here, not copied from any question paper. Treat them as a guide to what examiners emphasise, not as a prediction of the exact paper.

About the marks and timings. The weightage and study-time figures at the start of the unit are practical estimates based on the 08-hour teaching allocation the unit receives and on the 70-mark end-semester structure. Your college may distribute marks slightly differently.

UNIT I

Fundamentals of Photonics

08 teaching hours · Course Outcome CO1

Photonics is the branch of engineering that treats light as a tool rather than as a subject of study. This unit has two halves that fit together neatly. The first half asks: *how do we make a beam of light that is intense, pure and perfectly directed?* — that is the laser, and holography is its most striking application. The second half asks: *once we have such a beam, how do we carry it from one place to another without losing it?* — that is the optical fibre, and fibre-optic communication is its most important application. Almost every question from this unit is a variation on one of those two questions.

Expected weightage	About 14 of 70 marks	**Typical marks**	12–16 marks	**Study time**	10–12 hours

✔ LEARNING OBJECTIVES — after this unit you should be able to

1. Distinguish clearly between absorption, spontaneous emission and stimulated emission, and state the condition needed for light amplification.
2. Explain population inversion, pumping, the active medium, active centres and the resonant cavity, and describe the job each one does.
3. List and justify the four characteristics that make laser light different from ordinary light.
4. Draw and explain the construction and working of a CO_2 laser, including its energy-level scheme.
5. Describe engineering applications of lasers in information technology, medicine and industry.
6. Explain how a hologram is recorded and reconstructed, and why it gives a true three-dimensional image.
7. Define critical angle, acceptance angle, acceptance cone and numerical aperture, and derive the expression for numerical aperture.
8. Classify optical fibres by mode and by index profile and compare them.
9. Define the attenuation coefficient, list the causes of attenuation and interpret the attenuation-versus-wavelength curve.
10. State the advantages of optical-fibre communication and solve numerical problems on fibre parameters.

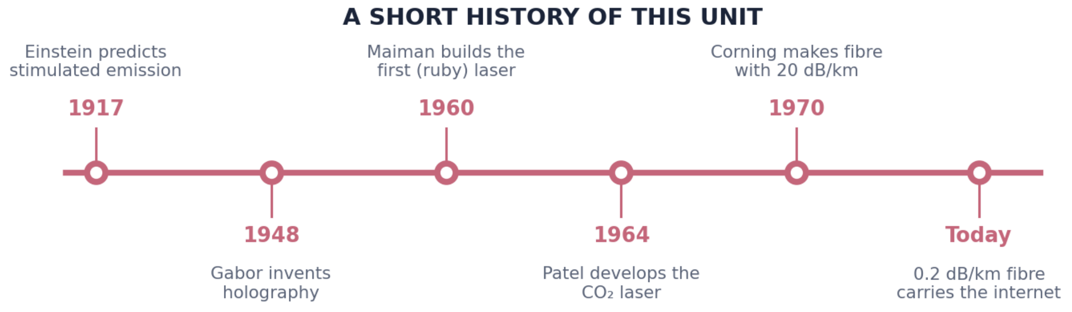


Figure 1.0 — The century of work behind this unit.

Notice how long the gap is between Einstein's prediction of stimulated emission in 1917 and Maiman's first working laser in 1960. The physics was understood decades before the engineering caught up — and the same is true of the optical fibre, which only became practical when Corning brought the loss below 20 dB/km in 1970.

1.1 Spontaneous and Stimulated Emission

◆ **DEFINITION**

Absorption — when a photon of energy exactly equal to the gap between two energy levels falls on an atom in the lower level, the atom absorbs the photon and jumps to the higher level.

Spontaneous emission — an atom sitting in a higher energy level falls back to a lower level entirely on its own, without any outside trigger, and gives out a photon of energy $h\nu = E_2 - E_1$.

Stimulated emission — an atom in a higher energy level is struck by a photon of energy exactly $h\nu = E_2 - E_1$ and is forced to fall to the lower level, giving out a second photon that is *identical* to the first in frequency, phase, direction and polarisation.

Basic concept — start from nothing

An atom can only hold certain fixed amounts of energy. Picture these as shelves at fixed heights: an electron can sit on the ground shelf or on a higher shelf, but never in between. Moving between shelves always involves a photon — a packet of light energy. Three things can happen, and only three.

- The electron can be **pushed up** a shelf by swallowing a photon. That is absorption. Light goes in; no light comes out.
- The electron can **fall down** by itself after a short wait. That is spontaneous emission. No light goes in; one photon comes out, in a random direction, at a random moment.
- The electron can be **knocked down** early by a passing photon. That is stimulated emission. One photon goes in; *two* come out, and they are exact twins.

The third process is the whole of laser physics. It is the only one of the three that increases the number of photons while keeping them identical — that is what “amplification of light” means.

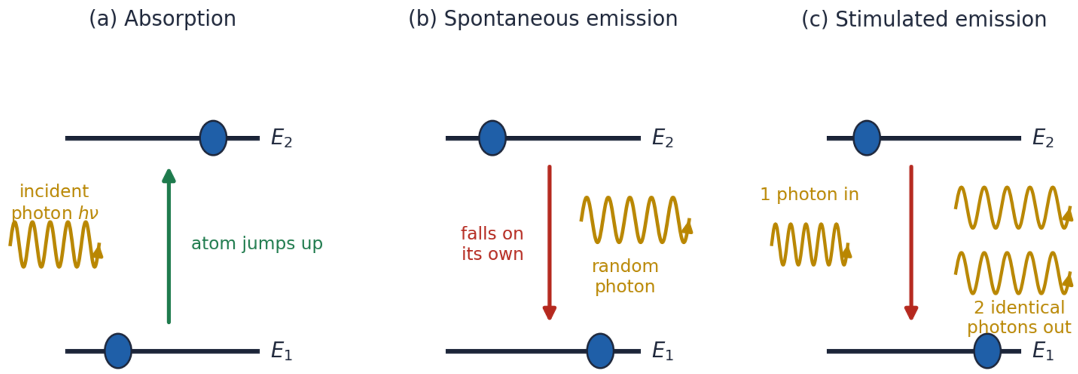


Figure 1.1 — The three radiative processes between two energy levels E_1 and E_2.

Notice the difference between panels (b) and (c). In spontaneous emission the photon leaves in a random direction with a random phase, so a lamp built on spontaneous emission gives incoherent light spraying out in all directions. In stimulated emission the new photon copies the incoming photon exactly, so the two travel together as one wave. Repeat this millions of times and you have a laser beam.

Detailed explanation

Einstein's coefficients — putting numbers on the three processes

In 1917 Einstein showed that the three processes are linked. If N_1 and N_2 are the numbers of atoms per unit volume in the lower and upper levels, and ρ(ν) is the energy density of radiation at frequency ν, then the rate of each process is:

$$R_{abs} = B_{12} N_1 \rho(\nu) \quad \cdot \quad R_{spon} = A_{21} N_2 \quad \cdot \quad R_{stim} = B_{21} N_2 \rho(\nu)$$

A_{21}, B_{12} and B_{21} are Einstein's coefficients

Meaning of each symbol

Symbol	What it means	SI unit
A_{21}	Probability per second of spontaneous emission — depends only on the atom, not on the light present.	s^{-1}
B_{12}	Coefficient of stimulated absorption.	$m^3 J^{-1} s^{-2}$
B_{21}	Coefficient of stimulated emission.	$m^3 J^{-1} s^{-2}$
ρ(ν)	Energy density of the radiation field at frequency ν.	$J m^{-3} Hz^{-1}$
N_1, N_2	Number of atoms per unit volume in the lower and upper levels.	m^{-3}

Two results follow, and both are worth remembering. First, $B_{12} = B_{21}$ for levels of equal degeneracy — an atom is exactly as willing to be stimulated down as it is to absorb. Second, the ratio A_{21}/B_{21} is proportional to ν^3. That second result explains a great deal: at high frequencies spontaneous emission wins easily, which is why building an X-ray laser is far harder than building an infrared one.

Why stimulated emission normally loses

Compare the rate of stimulated emission with the rate of absorption. Since $B_{12} = B_{21}$, the ratio is simply N_2/N_1. In ordinary matter at thermal equilibrium the Boltzmann distribution gives

$$N_2 / N_1 = \exp[-(E_2 - E_1) / k_B T]$$

the Boltzmann ratio — always less than 1 at any positive temperature

For a visible transition ($E_2 - E_1 \approx 2$ eV) at room temperature, $k_B T \approx 0.026$ eV, so the ratio is about e^{-77} — effectively zero. Almost every atom is in the lower level, so almost every photon that arrives is absorbed. Light passing through ordinary matter gets weaker, never stronger. To make a laser we have to break this equilibrium deliberately.

Worth memorising

The key comparison table. If you learn one table from this topic, learn this one — it is asked directly, almost every year.

Point of comparison	Spontaneous emission	Stimulated emission
Trigger	None — happens on its own	An incoming photon of exactly the right energy
Photons out per event	One	Two (the trigger plus a new twin)
Direction	Random	Same as the incoming photon
Phase	Random	Same as the incoming photon
Polarisation	Random	Same as the incoming photon
Nature of light produced	Incoherent, spreads out	Coherent, highly directional
Rate depends on	N_2 only	N_2 and the radiation density $\rho(\nu)$
Controllable?	No — set by the atom's lifetime	Yes — increase $\rho(\nu)$ and the rate rises
Example	Sodium lamp, candle flame, LED	Laser

Table 1.1 — Spontaneous versus stimulated emission.

⚙ REAL-LIFE APPLICATIONS

- **Fluorescent tube lights and LEDs** work on spontaneous emission — that is exactly why their light is soft and spreads over a room.
- **Every laser**, from the pointer in a lecture hall to the fibre-laser cutting sheet metal in a factory, works on stimulated emission.
- **Optical amplifiers** in undersea communication cables (erbium-doped fibre amplifiers) amplify the signal directly by stimulated emission, without converting it back to electricity.

✔ REMEMBER THESE POINTS

- Three processes only: absorption, spontaneous emission, stimulated emission.
- Stimulated emission is the *only* one that multiplies identical photons — it is the basis of the laser.
- $B_{12} = B_{21}$; $A_{21}/B_{21} \propto \nu^3$.
- At thermal equilibrium $N_2 \ll N_1$, so absorption always wins — equilibrium must be broken.
- "LASER" = Light Amplification by Stimulated Emission of Radiation.

⚠ COMMON MISTAKES

- Writing that stimulated emission produces "one photon". It produces **two** — the original one is not consumed.
- Saying the emitted photon is "similar to" the incoming one. Examiners want **identical in frequency, phase, direction and polarisation** — all four.
- Confusing stimulated emission with stimulated *absorption*. Both need an incoming photon, but one starts from the lower level and one from the upper.
- Forgetting that A_{21} has different units from B_{21} — a common trap in a "state the units" question.

★ EXAM TIP

- A 4-mark question almost always means: define all three processes, then draw Figure 1.1. Draw the diagram first — it organises your answer.
- A 6-mark question usually adds the comparison table. Give at least five rows.
- If asked "why is stimulated emission necessary for a laser?", the one-line answer is: *it is the only process that produces coherent photons in a controlled direction, so it is the only one that can amplify a beam*.

1.2 Population Inversion and Pumping

◆ DEFINITION

Population inversion is the non-equilibrium condition in which the number of atoms in a higher energy level exceeds the number in a lower energy level, i.e. $N_2 > N_1$.

Pumping is the process of supplying energy to the active medium from outside in order to raise atoms to the upper level and so create and maintain population inversion.

A **metastable state** is an excited level with an unusually long lifetime (typically 10^{-3} s instead of the usual 10^{-8} s), which allows atoms to accumulate there.

Basic concept

Think of a crowded staircase. Normally almost everyone is standing on the ground floor, because that costs the least energy — that is the Boltzmann distribution. Population inversion means deliberately getting more people onto an upper landing than are left on the floor below. It is an unnatural, forced situation, and it will collapse the moment you stop pushing. The "pushing" is pumping.

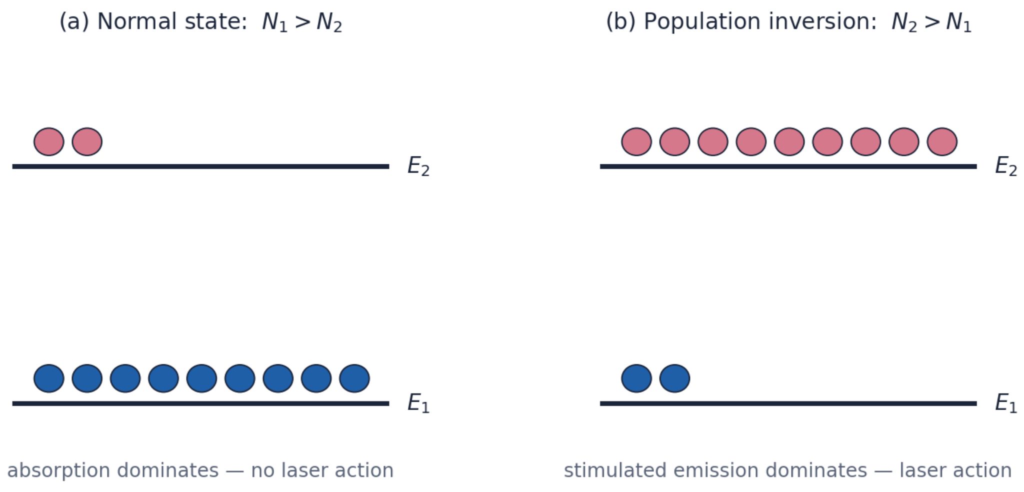


Figure 1.2 — Normal thermal population compared with population inversion.

In (a) the lower level is crowded, so an incoming photon will almost certainly meet a lower-level atom and be absorbed — the beam is attenuated. In (b) the upper level is crowded, so the same photon will almost certainly meet an upper-level atom and trigger stimulated emission — the beam is amplified. Population inversion is the switch that turns a medium from an absorber into an amplifier.

Detailed explanation — why two levels are never enough

You might think that shining a very bright light on a two-level system would eventually push more than half the atoms upstairs. It cannot. The same light that pumps atoms up also stimulates them back down, and since $B_{12} = B_{21}$ the two rates become equal as soon as $N_2 = N_1$. The best a two-level system can ever reach is equal populations, called *saturation*. This is a favourite short-answer question, so remember the reason: *the pump is just as good at bringing atoms down as it is at taking them up.*

The way round the problem is to use a third (or fourth) level, so that the level being pumped is not the level from which the laser transition starts.

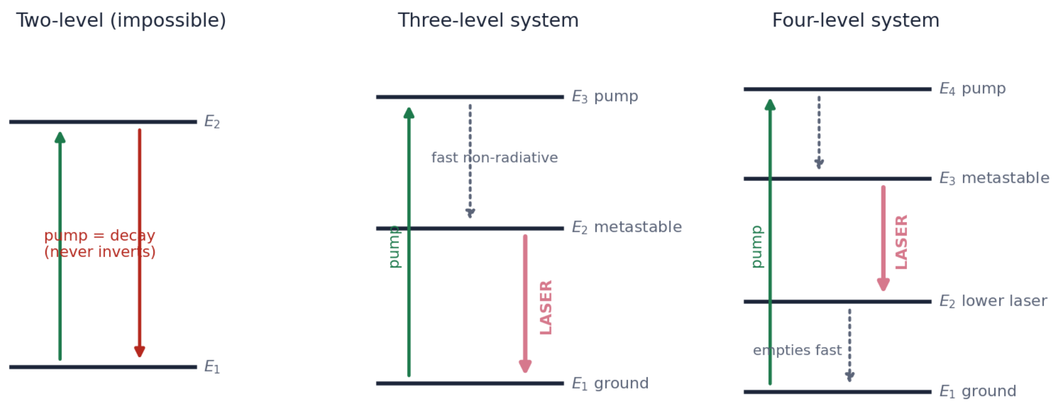


Figure 1.3 — Two-, three- and four-level laser schemes.

In the three-level scheme (used by the ruby laser) atoms are pumped to E_3, fall quickly and without radiating to the metastable level E_2, and pile up there; the laser transition then runs from

E_2 down to the ground state E_1. Because the lower laser level *is* the ground state, more than half of all the atoms must be lifted before inversion begins — so three-level lasers need very hard pumping and usually run in pulses. In the four-level scheme (used by the CO_2 and He–Ne lasers) the lower laser level E_2 is well above the ground state and empties itself almost instantly, so it is nearly always empty. Inversion is then achieved with very little pumping, which is why four-level lasers can run continuously.

Methods of pumping

Pumping method	How energy is supplied	Typical lasers
Optical pumping	Intense light from a flash lamp, an arc lamp or another laser is absorbed by the active medium.	Ruby, Nd:YAG, dye lasers
Electrical discharge	A high voltage across a gas accelerates electrons, which excite the atoms or molecules by collision.	CO_2, He–Ne, argon-ion lasers
Direct electrical injection	Current is forced across a p-n junction; electrons and holes recombine and emit light.	Semiconductor (diode) lasers
Chemical pumping	The energy released by a chemical reaction leaves the products in an excited state.	HF and DF chemical lasers
Thermal / gas-dynamic	A hot gas is expanded suddenly through a nozzle so that the upper level cools more slowly than the lower level.	Gas-dynamic CO_2 lasers

Table 1.2 — The common pumping methods.

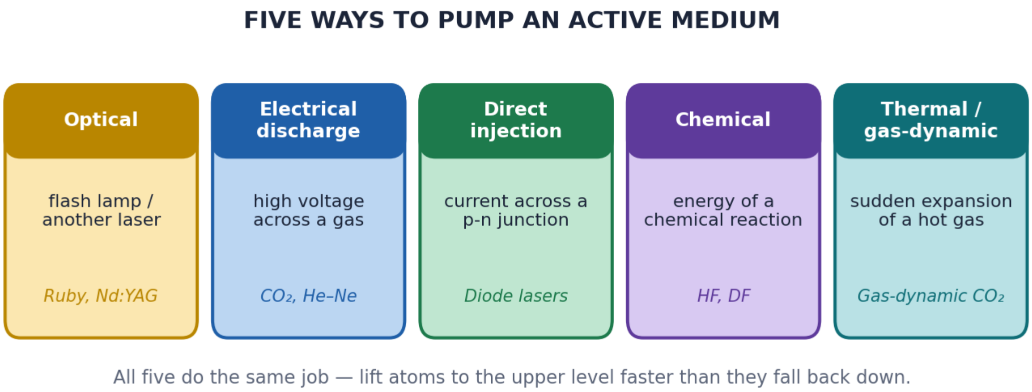


Figure 1.3a — The five pumping methods at a glance.

> **● IMPORTANT NOTE**
> **The role of the metastable state.** Ordinary excited states live for about 10^{-8} s — far too short for atoms to accumulate. A metastable state lives about 10^{-3} s, roughly a hundred thousand times longer, because the transition down from it is “forbidden” by the selection rules of quantum mechanics. That long wait is what allows the upper level to fill up faster than it empties. **No metastable state, no population inversion, no laser.**

✔ REMEMBER THESE POINTS

- Population inversion: $N_2 > N_1$ — an unnatural, non-equilibrium state.
- It is the necessary condition for light amplification.
- A two-level system can never invert; it saturates at $N_2 = N_1$.
- Three-level lasers need > 50 % of atoms excited → hard pumping → usually pulsed.
- Four-level lasers invert easily → continuous-wave operation.
- A metastable state (lifetime ≈ 10^{-3} s) is essential.

⚠ COMMON MISTAKES

- Writing "population inversion means all atoms are in the upper state". It only means *more* atoms are up than down for the pair of levels concerned.
- Saying a two-level laser is "difficult". It is **impossible**, and you should say why.
- Mixing up pumping with the laser transition. Pumping is the energy input; the laser transition is the energy output.
- Forgetting to mention the metastable level when explaining three- and four-level schemes — it is usually worth a mark on its own.

★ EXAM TIP

- "Explain population inversion" nearly always carries the follow-up "...and explain why a two-level system cannot achieve it." Prepare both together.
- Draw the three-level and four-level diagrams side by side; the comparison earns marks quickly.
- If asked to compare three- and four-level systems, the crisp answer is: *the lower laser level is the ground state in one and an empty excited state in the other*.

1.3 Active Medium, Active Centres and the Resonant Cavity

◆ DEFINITION

The **active medium** is the material — solid, liquid, gas or semiconductor — inside which population inversion is created and stimulated emission takes place.

Active centres are the particular atoms, ions or molecules within that medium that actually take part in the laser transition. In ruby, for example, the medium is aluminium oxide but the active centres are the chromium ions (Cr^{3+}) present as an impurity.

The **resonant cavity** (or optical resonator) is the pair of parallel mirrors placed on either side of the active medium; it makes the light pass through the medium many times and selects the direction and wavelength of the output.

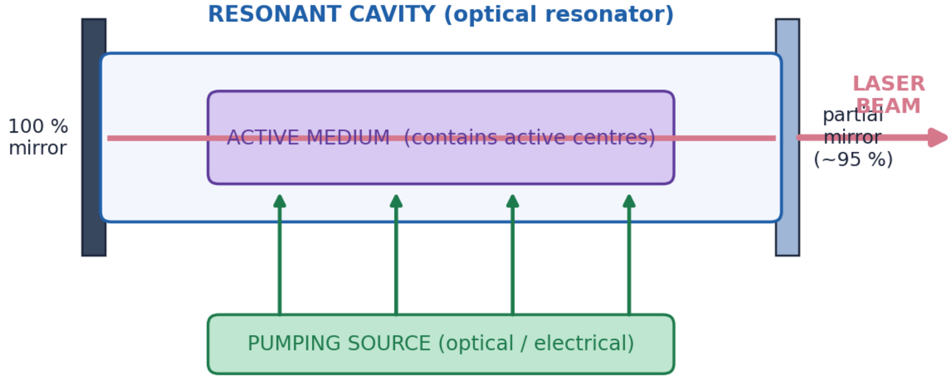


Figure 1.4 — The three essential components of every laser.

Every laser you will ever meet has these three parts and no others: an active medium containing active centres, a pumping source to invert it, and a resonant cavity to build the beam. If an examination question asks for the "basic components of a laser", this single figure is the complete answer.

Detailed explanation — what the cavity actually does

It is tempting to think the mirrors are there simply to make the beam come out of one end. They do four separate jobs, and a good answer lists all four.

1. **They provide optical feedback.** A photon that leaves the medium is sent back through it again and again, so a single spontaneous photon can trigger an avalanche of millions of stimulated photons. Without feedback you would have an amplifier, not an oscillator.
2. **They select the direction.** Only photons travelling almost exactly along the axis survive many round trips; anything at an angle walks off the mirrors within a few passes. This is what makes a laser beam so narrow.
3. **They select the wavelength.** The cavity supports only those wavelengths for which a whole number of half-wavelengths fits between the mirrors — the longitudinal modes. This narrows the line and improves monochromaticity.
4. **They couple the light out.** One mirror is made partially reflecting (typically 95–99 %) so that a controlled fraction of the circulating power escapes as the useful beam.

The standing-wave (longitudinal mode) condition

For a wave to survive many round trips it must reproduce itself exactly after each trip. If L is the separation of the mirrors and λ is the wavelength inside the cavity, this requires:

$$L = n\lambda / 2 \quad \rightarrow \quad \nu_n = n c / (2L)$$

n = 1, 2, 3, … is the mode number

Meaning of each symbol

Symbol	What it means	SI unit
L	Separation between the two mirrors (cavity length).	m
λ	Wavelength of the light inside the cavity.	m
n	A positive integer — the number of half-wavelengths that fit.	dimensionless
ν_n	Frequency of the n-th longitudinal mode.	Hz
c	Speed of light in the medium.	$m\ s^{-1}$

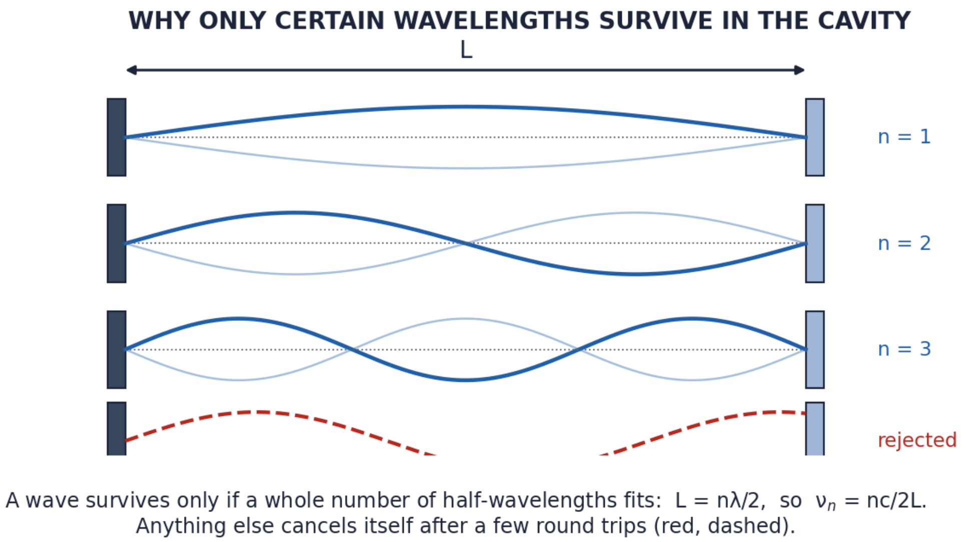


Figure 1.4a — Why the cavity allows only certain wavelengths.

The frequency spacing between neighbouring modes is therefore $\Delta\nu = c / 2L$. For a cavity 30 cm long this is about 500 MHz — very small compared with an optical frequency of about 5×10^{14} Hz, which is why a laser line is so extraordinarily narrow.

Threshold condition

Laser action begins only when the gain from stimulated emission in one round trip exceeds all the losses — mirror transmission, scattering, absorption and diffraction. This break-even point is called the **threshold**. Below threshold the device is just a bright lamp; above threshold the output rises steeply and almost linearly with pump power.

⚙ REAL-LIFE APPLICATIONS

- The choice of active medium fixes the wavelength, and therefore the application: CO_2 (10.6 µm) for cutting metal, Nd:YAG (1.064 µm) for engraving, semiconductor diodes (0.85–1.55 µm) for fibre communication, argon-ion (0.488 µm) for retinal surgery.
- Cavity length is used to tune fibre-laser and diode-laser systems in optical communication.
- Ring cavities (three or four mirrors, no standing wave) are used in laser gyroscopes for aircraft navigation.

✔ REMEMBER THESE POINTS

- Three essential parts: active medium (with active centres), pumping source, resonant cavity.
- Cavity jobs: feedback, direction selection, wavelength selection, output coupling.
- Mode condition $L = n\lambda/2$; mode spacing $\Delta\nu = c/2L$.
- Laser action starts only above threshold — gain per round trip > loss per round trip.
- One mirror is 100 % reflecting, the other 95–99 %.

⚠ COMMON MISTAKES

- Calling the whole tube the “active centre”. The medium is the tube's contents; the active centres are the specific species that lase.
- Saying both mirrors are fully reflecting — then no light could ever leave.
- Writing $L = n\lambda$ instead of $L = n\lambda/2$. The factor of two comes from the wave having to fit an integer number of *half*-wavelengths.

★ EXAM TIP

- A labelled Figure 1.4 with one line of explanation per part is a complete 4-mark answer.
- If the question says “functions of the resonant cavity”, give all four functions as numbered points — one mark each.

1.4 Characteristics of Laser Light

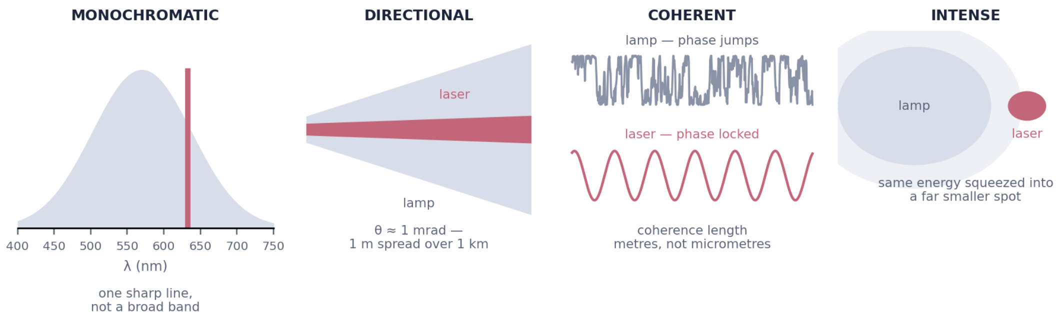


Figure 1.4b — Laser light compared with the light of an ordinary lamp.

Four properties separate laser light from the light of a lamp. They all follow from the same cause — the photons are produced by stimulated emission inside a resonant cavity — but examiners expect them to be listed and explained separately.

Characteristic	What it means	Why a laser has it	Typical figure
Monochromaticity	The beam contains an extremely narrow band of wavelengths — practically a single colour.	All photons come from one transition between two sharp energy levels, and the cavity further narrows the line.	$\Delta\lambda \approx 10^{-6}$ nm for a stabilised He–Ne, against ≈ 100 nm for a lamp
Coherence	*Temporal*: the phase stays predictable over a long time. *Spatial*: the phase is the same across the whole beam width.	Stimulated emission copies the phase of the triggering photon exactly, so every photon is in step.	Coherence length of several hundred metres, against ≈ 1 µm for a bulb
Directionality	The beam spreads very little as it travels.	Only rays travelling along the cavity axis survive repeated reflection; everything else is lost.	Divergence ≈ 1 milliradian — about 1 m spread over 1 km
High intensity / brightness	Enormous power concentrated into a very small area and a very narrow solid angle.	The energy is not spread over all directions or all wavelengths, and it can be focused to a spot a few wavelengths across.	Focused intensities above 10^{12} W m^{-2} are routine

Table 1.3 — The four characteristics of laser light.

Divergence — the formula you will be asked to use

If a beam of initial diameter d_1 becomes d_2 after travelling a distance $(x_2 - x_1)$, the full-angle divergence is:

$$\theta = (d_2 - d_1) / (x_2 - x_1) \quad \text{(in radian)}$$

Meaning of each symbol

Symbol	What it means	SI unit
θ	Full-angle beam divergence — how fast the beam widens.	rad (or mrad)
d_1, d_2	Beam diameters at the two positions.	m
$x_2 - x_1$	Distance between the two positions.	m

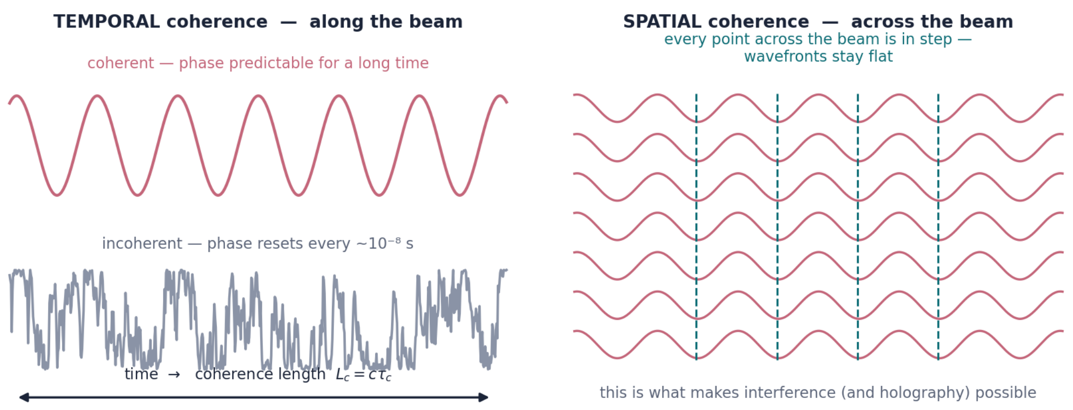


Figure 1.4c — The two kinds of coherence.

IMPORTANT NOTE

Temporal and spatial coherence are different things. Temporal coherence is about *when* — how long the wave keeps a predictable phase, measured by the coherence length $L_c = c \cdot \tau_c$. Spatial coherence is about *where* — whether two points across the beam are in step. A question asking about "coherence" expects both to be mentioned.

REMEMBER THESE POINTS

- Four characteristics: monochromaticity, coherence, directionality, high intensity.
- Coherence has two kinds: temporal (along the beam) and spatial (across the beam).
- Coherence length $L_c = c\,\tau_c$.
- Divergence $\theta = (d_2 - d_1)/(x_2 - x_1)$, typically ≈ 1 mrad.
- All four properties trace back to stimulated emission inside a cavity.

EXAM TIP

- This topic appears both as a 4-mark theory question and as a small numerical on divergence. Prepare both.
- Always give a number alongside each property — examiners reward the comparison with an ordinary lamp.

1.5 CO Laser — Construction and Working

DEFINITION

The **CO_2 laser** is a four-level molecular gas laser in which the laser transition takes place between *vibrational* energy levels of the carbon dioxide molecule, producing infrared radiation at 10.6 μm with an efficiency of about 20–30 % — the highest of any conventional laser.

Basic concept — why a molecule instead of an atom

In the lasers you have met so far the energy levels belong to electrons orbiting an atom. A molecule has an extra way of storing energy: its atoms can vibrate against one another. These vibrational levels are much closer together than electronic levels, so the photon emitted has much less energy and a much longer wavelength — in the infrared. That is why the CO_2 laser emits at 10.6 μm rather than in the visible.

A CO_2 molecule is linear, O=C=O, and it has three ways to vibrate:

- **Symmetric stretch** — both oxygen atoms move out and in together, the carbon stays still. Denoted (100).
- **Bending** — the molecule bends out of a straight line. Denoted (010).
- **Asymmetric stretch** — one oxygen moves out while the other moves in. Denoted (001). This is the upper laser level.

Construction

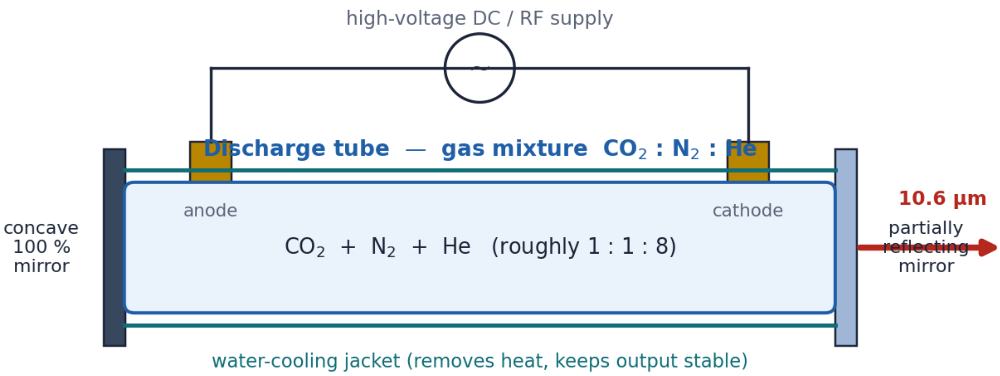


Figure 1.5 — Construction of a CO_2 laser.

The discharge tube is typically 1–3 m long and a few centimetres in diameter. Since the output is at 10.6 µm, ordinary glass is opaque to it — the windows and output mirror must be made of germanium, zinc selenide or gallium arsenide. This is a favourite one-mark detail.

- **Discharge tube** — a long glass or quartz tube containing the gas mixture at a low pressure of a few torr.
- **Gas mixture** — carbon dioxide, nitrogen and helium, in a ratio of roughly 1 : 1 : 8. Each gas has a specific job (see below).
- **Electrodes** — a pair of electrodes at the ends of the tube, connected to a high-voltage DC or radio-frequency supply that maintains the discharge.
- **Resonant cavity** — a fully reflecting concave mirror at one end and a partially reflecting mirror of germanium or zinc selenide at the other.
- **Water-cooling jacket** — surrounds the tube. Cooling is essential: if the gas gets hot the lower laser levels stay populated and the inversion collapses.

The job of each gas — a standard examination question

Gas	Proportion	What it does
Carbon dioxide (CO_2)	≈ 1 part	The active centre. Its asymmetric-stretch level (001) is the upper laser level; the 10.6 µm and 9.6 µm transitions start here.
Nitrogen (N_2)	≈ 1 part	The energy carrier. Electrons in the discharge excite N_2 to its first vibrational level, which is metastable and — crucially — almost exactly in resonance with the CO_2 (001) level. Energy passes across by collision, filling the upper laser level efficiently.
Helium (He)	≈ 8 parts	The housekeeper. It has high thermal conductivity, so it carries heat to the tube wall and keeps the gas cool; it also de-excites the lower laser levels rapidly by collision, keeping them empty.

Table 1.4 — Why the CO_2 laser needs three gases.

Working — the energy-level story

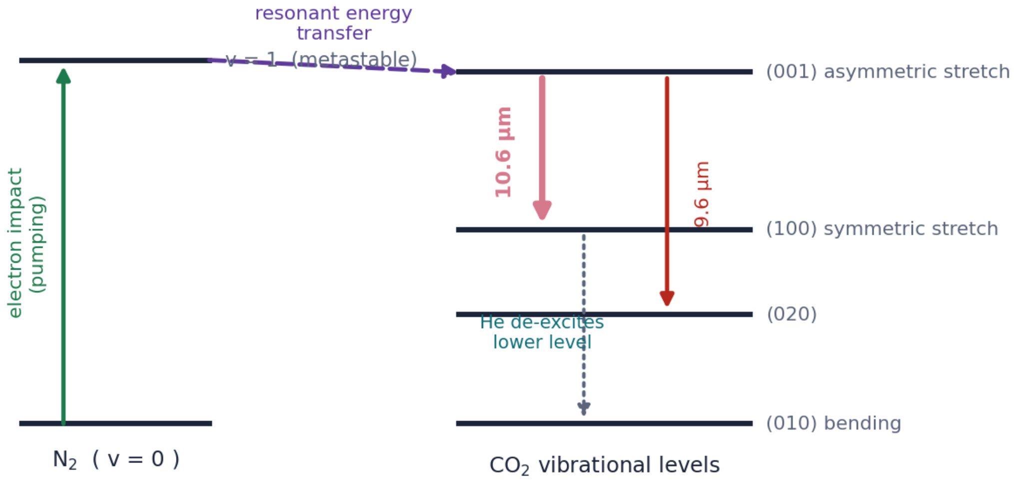


Figure 1.6 — Energy-level scheme of the CO_2 laser.

Follow the arrows in order: electrons excite N_2; N_2 passes its energy resonantly to the CO_2 (001) level; the laser transition (001) → (100) gives 10.6 µm; a weaker transition (001) → (020) gives 9.6 µm; helium collisions empty the lower levels back to the ground state.

1. A high voltage across the tube produces a glow discharge. Free electrons are accelerated and collide with the gas molecules.
2. The electrons excite nitrogen molecules to the v = 1 vibrational level. This level is **metastable**, because a symmetric molecule like N_2 cannot radiate a vibrational photon — so the energy is stored.
3. Excited N_2 molecules collide with ground-state CO_2 molecules. Because the N_2 (v = 1) energy and the CO_2 (001) energy are almost identical, the transfer is **resonant** and highly efficient. This is the main pumping route.
4. The CO_2 (001) level fills up while the (100) and (020) levels stay nearly empty — **population inversion** is established.
5. Stimulated emission from (001) to (100) gives the main output at **10.6 µm**; from (001) to (020) gives a weaker line at **9.6 µm**.
6. Collisions with helium atoms rapidly drop the molecules from (100) and (020) to the (010) bending level and then to the ground state, keeping the lower laser level empty so that the inversion is maintained continuously.
7. The photons bounce between the mirrors, build up, and a fraction emerges through the partially reflecting mirror as a continuous infrared beam.

IMPORTANT NOTE

Why the efficiency is so high. An ordinary laser wastes most of the pump energy lifting electrons to high electronic levels and then throwing that energy away as heat. In the CO_2 laser the pump energy goes into vibration, the energy gap between the pump level and the upper laser level is tiny, and the transfer from N_2 is resonant. Very little is wasted, so 20–30 % of the electrical input comes out as light — against about 0.1 % for a He–Ne laser.

Property	Typical value
Type	Four-level molecular gas laser
Active centre	CO_2 molecule (vibrational levels)
Pumping	Electrical discharge, with resonant transfer from N_2
Output wavelength	10.6 µm (main), 9.6 µm (secondary) — infrared
Mode of operation	Continuous wave or pulsed
Output power	A few watts to tens of kilowatts
Efficiency	About 20–30 %
Mirror / window material	Germanium, ZnSe or GaAs (glass is opaque at 10.6 µm)

Table 1.5 — CO_2 laser at a glance.

⚙ REAL-LIFE APPLICATIONS

- **Industrial cutting and welding** of steel, aluminium and plastics — the high power and good absorption of 10.6 µm by most materials make it the workhorse of laser machining.
- **Laser marking and engraving** on wood, leather, acrylic and glass.
- **Surgery** — the beam is strongly absorbed by water, and since tissue is mostly water it vaporises a very thin layer while sealing blood vessels. Used in dermatology and in ENT surgery.
- **Remote sensing and LIDAR** — 10.6 µm sits in an atmospheric transmission window.
- **Isotope separation** and materials research.

✔ REMEMBER THESE POINTS

- CO_2 laser = four-level **molecular** gas laser; laser levels are vibrational.
- Gas ratio $CO_2 : N_2 : He \approx 1 : 1 : 8$.
- N_2 (v = 1) is metastable and resonant with CO_2 (001) — this is the pumping route.
- Main output 10.6 µm; secondary 9.6 µm.
- Helium cools the gas and empties the lower laser level.
- Efficiency 20–30 % — the highest of the common lasers.
- Optics must be germanium or ZnSe, not glass.

⚠ COMMON MISTAKES

- Writing that the laser transition is between *electronic* levels. It is between **vibrational** levels — this is the defining feature.
- Giving the wavelength as 10.6 nm or 1.06 µm. It is **10.6 µm**. (1.064 µm is the Nd:YAG laser — a classic mix-up.)
- Forgetting helium, or giving it only the cooling role. It also de-excites the lower laser level.
- Omitting the water jacket from the diagram — it is a labelled part and carries a mark.
- Saying the mirrors are ordinary glass.

> **★ EXAM TIP**
> - This is one of the most frequently asked long questions in the unit. A full answer = labelled construction diagram + energy-level diagram + the role of each gas + the numbered working steps.
> - Draw both diagrams even if the question only says “explain” — they save you writing and gain marks.
> - Keep the ratio 1 : 1 : 8 and the figure 10.6 μm at your fingertips; they are the two facts most often asked for directly.

1.6 Engineering Applications of Lasers

The syllabus asks for applications in three areas — information technology, medicine and industry. Learn two or three from each area with one line of physics attached to every one. An application without its reason earns half the mark.

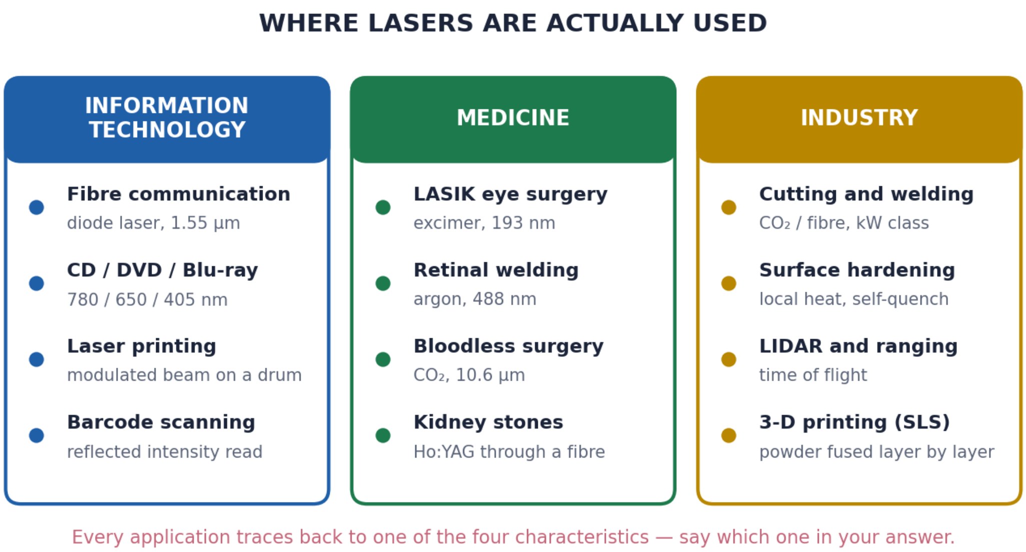


Figure 1.6 — Laser applications across the three areas the syllabus names.

A. Information technology and communication

Application	The physics behind it
Optical fibre communication	A semiconductor laser is modulated at gigahertz rates; its monochromatic, coherent output couples efficiently into a single-mode fibre and suffers little dispersion.
Optical data storage (CD, DVD, Blu-ray)	The beam is focused to a spot whose size is limited by diffraction to about λ. Shorter wavelength → smaller pit → more data. CD uses 780 nm, DVD 650 nm, Blu-ray 405 nm.
Laser printing and barcode scanning	A modulated beam writes a charge pattern on a photosensitive drum; in scanners the reflected intensity is read as a digital signal.
Free-space optical links and LiFi	The low divergence of the beam allows point-to-point links without cable.
Holographic and quantum-key data security	Coherence allows the phase of the beam to carry information.

B. Medicine

Application	The physics behind it
Laser eye surgery (LASIK)	An excimer laser at 193 nm breaks molecular bonds directly (photo-ablation) and removes corneal tissue a fraction of a micrometre at a time without heating the surroundings.
Retinal detachment repair (photocoagulation)	An argon laser at 488 nm is strongly absorbed by haemoglobin, so it spot-welds the retina back to the underlying layer.
Bloodless surgery	A CO_2 beam vaporises tissue and simultaneously cauterises small blood vessels, so the cut hardly bleeds.
Kidney-stone and gallstone fragmentation	A pulsed Ho:YAG beam delivered through a fibre shatters the stone by generating a shock wave.
Dermatology, tattoo removal and dentistry	Selective photothermolysis — the wavelength is chosen so that only the target pigment absorbs strongly.
Diagnostics: endoscopy, optical coherence tomography, flow cytometry	Coherence allows interferometric depth imaging with micrometre resolution.

C. Industry

Application	The physics behind it
Cutting, drilling and welding	Focused intensity above 10^6 W cm^{-2} melts and vaporises metal at the focal spot; the heat-affected zone stays small because the energy is delivered so locally.
Surface hardening and cladding	Controlled heating and rapid self-quenching change the microstructure of the surface only.
Precision measurement and alignment	Interferometry measures displacement to a fraction of a wavelength; laser levels align tunnels, pipelines and machine beds.
Distance measurement (LIDAR, range finders)	Time-of-flight or phase-shift measurement of a returned pulse.

Application	The physics behind it
3-D printing (selective laser sintering)	The beam fuses metal or polymer powder layer by layer.
Non-destructive testing and laser marking	Speckle interferometry reveals sub-micrometre surface strain.

✔ REMEMBER THESE POINTS

- IT: fibre communication, optical storage, printing, scanning.
- Medicine: LASIK, photocoagulation, bloodless surgery, lithotripsy, dermatology.
- Industry: cutting, welding, hardening, measurement, LIDAR, 3-D printing.
- Every application traces back to one of the four characteristics — say which one.

★ EXAM TIP

- Structure your answer under the three headings the syllabus uses. Examiners look for that structure.
- Give the laser type and wavelength where you can — “CO_2 at 10.6 μm for cutting” scores better than “lasers are used for cutting”.

1.7 Holography — Recording, Reconstruction and Applications

◆ DEFINITION

Holography is a two-step photographic technique, invented by Dennis Gabor in 1948, that records the *amplitude and the phase* of a light wave reflected from an object, and later reconstructs that wave so that a genuine three-dimensional image is seen.

A **hologram** is the recording medium after exposure: it carries a fine interference pattern, not a picture.

Basic concept — why an ordinary photograph is flat

A photographic film responds only to intensity, and intensity is the square of the amplitude. All the information about phase — which is what tells your eye how far away each point of the object is — is thrown away the instant the picture is taken. That is why a photograph is flat, and why moving your head does not let you see round the edge of an object in it.

Holography solves the problem with a trick borrowed from interference. Phase differences cannot be recorded directly, but they *can* be converted into intensity differences by making the object wave interfere with a clean reference wave. The film then records a set of fine fringes whose spacing and contrast encode both the amplitude and the phase. The laser is essential here: only a coherent source can produce stable interference between two beams that have travelled different distances.

Step 1 — Recording the hologram

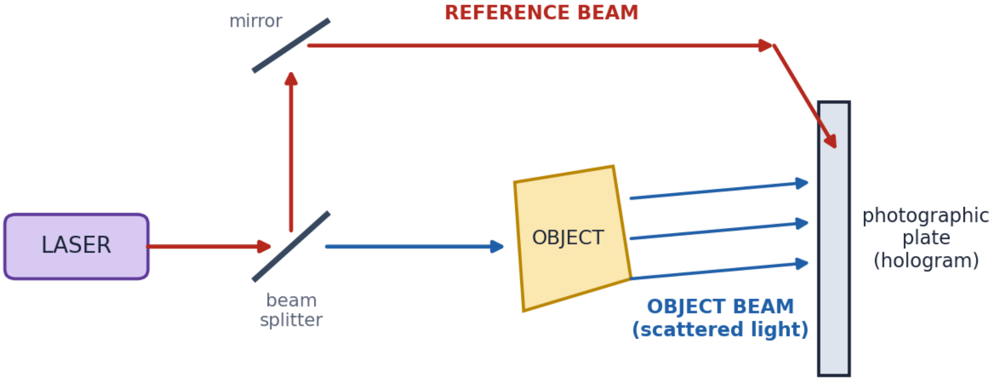


Figure 1.7 — Recording a hologram.

The beam splitter divides one laser beam into two, so both parts are automatically coherent with each other. One part illuminates the object and is scattered onto the plate — the *object beam*. The other reaches the plate directly — the *reference beam*. The plate records their interference pattern.

1. A laser beam is split into two by a beam splitter.
2. The **reference beam** is directed by a mirror straight onto the photographic plate.
3. The **object beam** illuminates the object; light scattered from every point of the object falls on the same plate.
4. The two beams overlap on the plate and interfere. Because they came from the same laser they are mutually coherent, so the fringe pattern is stable.
5. The plate is exposed and developed. The result is a dense, fine interference pattern that looks like nothing at all to the naked eye — it is **not** a picture of the object.

If the object wave is O and the reference wave is R, the plate records an intensity

$$I = |O + R|^2 = |O|^2 + |R|^2 + O R^* + O^* R$$

the last two cross terms are the ones that carry the phase information

The first two terms are the ordinary intensities of the two beams. The two cross terms contain the product of the object wave with the reference wave, and it is these that store the phase. This is why the reconstruction produces two images.

Step 2 — Reconstructing the image

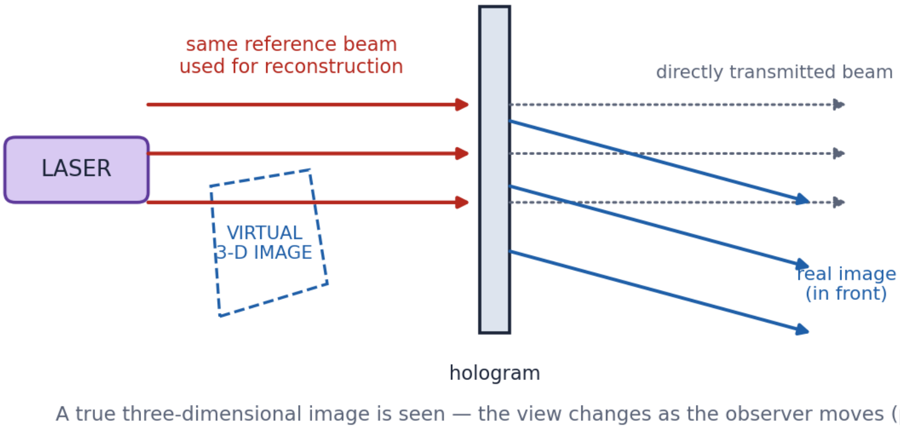


Figure 1.8 — Reconstructing the image from a hologram.

The developed hologram behaves as a complicated diffraction grating. When the reference beam alone illuminates it, diffraction regenerates a wave identical to the original object wave — the eye cannot tell the difference and sees a three-dimensional virtual image standing where the object used to be.

1. The developed hologram is illuminated by the **same reference beam** at the same angle, with the object removed.
2. The fringe pattern diffracts the light. Part of it passes straight through (the zero order).
3. One diffracted order reconstructs a wave that is an exact copy of the original object wave. Looking through the plate, the observer sees a **virtual image** in the position where the object stood — with full depth and parallax.
4. The other diffracted order forms a **real image** in front of the plate, which can be caught on a screen. It is pseudoscopic — its depth is inverted.

IMPORTANT NOTE

The most remarkable property of a hologram. Break a hologram into pieces and each piece still reproduces the whole object — only the viewing angle is restricted and the image is less sharp. This is because every point of the object sent light to every point of the plate, so the information about the whole object is stored everywhere on the plate. Break an ordinary photograph and you get pieces of the picture. This point is asked very often as a short question.

Point	Ordinary photograph	Hologram
What is recorded	Intensity only	Intensity and phase
Source needed	Any light	Coherent (laser) light
Appearance before viewing	A recognisable picture	A meaningless fringe pattern
Image	Two-dimensional	Three-dimensional, with parallax
Effect of breaking it	You lose that part of the picture	Each fragment still shows the whole object

Point	Ordinary photograph	Hologram
Lens required	Yes	No
Viewing	Direct	Needs the reference beam for reconstruction

Table 1.6 — Photograph compared with hologram.

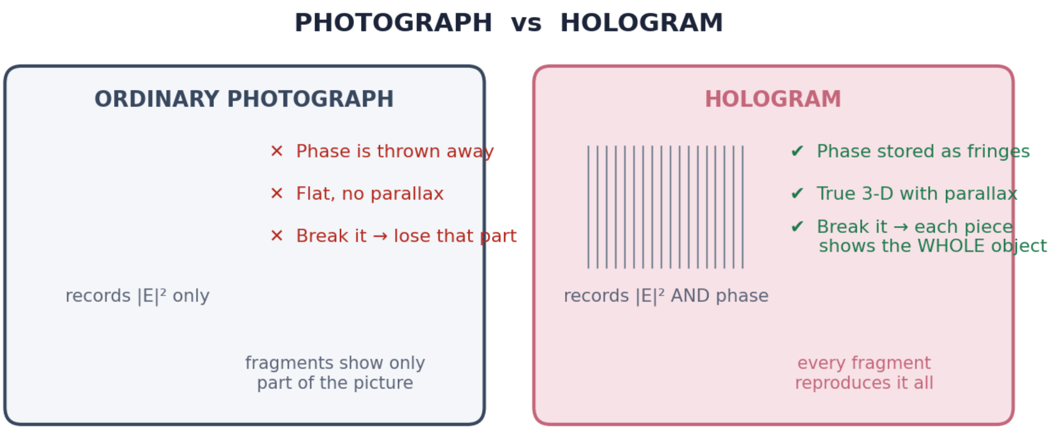


Figure 1.9 — Why a hologram is not simply a three-dimensional photograph.

⚙ REAL-LIFE APPLICATIONS

- **Security** — holograms on currency notes, credit cards, passports and product packaging, because they are extremely hard to copy.
- **Holographic data storage** — information is stored throughout the volume of a crystal rather than on a surface, giving very high density and page-at-a-time access.
- **Holographic interferometry** — a component is holographed before and after loading; the fringe pattern reveals deformations of a fraction of a micrometre. Widely used for non-destructive testing of tyres, turbine blades and aircraft panels.
- **Medical imaging** — three-dimensional records of CT and ultrasound data.
- **Holographic optical elements** — lenses, beam splitters and head-up displays in aircraft and cars.
- **Art, museums and advertising** — three-dimensional displays of fragile artefacts.

✔ REMEMBER THESE POINTS

- Holography records amplitude **and** phase; photography records amplitude only.
- Two steps: recording (object beam + reference beam interfere) and reconstruction (reference beam alone diffracts).
- A laser is essential — the two beams must be coherent.
- Reconstruction gives a virtual image (behind) and a real image (in front).
- Every fragment of a hologram reproduces the whole object.
- Invented by Dennis Gabor; Nobel Prize 1971.

⚠ COMMON MISTAKES

- Saying a hologram "is a 3-D photograph". It is an interference record — the image is three-dimensional, the hologram itself is not a picture.
- Forgetting that the **same** reference beam at the **same** angle must be used for reconstruction.
- Drawing the recording set-up without the beam splitter, or with two separate lasers. Two independent lasers are not mutually coherent.
- Mentioning only the virtual image. Both images are formed.

★ EXAM TIP

- "Explain recording and reconstruction of a hologram" is a standard 6-mark question — two labelled diagrams plus the numbered steps.
- The "broken hologram" property and the photograph-versus-hologram table are the two most common short questions.
- Mention holographic interferometry among the applications; it is the one with the strongest engineering flavour.

↻ QUICK REVISION BOX — Lasers and Holography

Definitions in one line each

- **Stimulated emission** — an incoming photon forces an excited atom down, producing a second identical photon.
- **Population inversion** — $N_2 > N_1$ between two levels.
- **Pumping** — supplying external energy to create inversion.
- **Active centre** — the species that actually lases.
- **Resonant cavity** — the mirror pair providing feedback, direction and wavelength selection.
- **Metastable state** — long-lived excited level ($\approx 10^{-3}$ s).
- **Hologram** — an interference record of amplitude and phase.

Formulas

$$N_2/N_1 = \exp[-(E_2-E_1)/k_BT] \quad \cdot \quad L = n\lambda/2 \quad \cdot \quad \Delta\nu = c/2L \quad \cdot \quad \theta = (d_2-d_1)/(x_2-x_1) \quad \cdot \quad E = hc/\lambda$$

Numbers to remember

- CO_2 laser: 10.6 μm main, 9.6 μm secondary, efficiency 20–30 %, gas ratio 1 : 1 : 8.
- Ordinary excited state $\approx 10^{-8}$ s; metastable $\approx 10^{-3}$ s.
- Typical divergence ≈ 1 mrad.
- Output mirror reflectivity ≈ 95–99 %.

Keywords examiners look for

- coherent · monochromatic · directional · high intensity · inversion · metastable · resonant transfer · feedback · threshold · reference beam · object beam · parallax

1.8 Optical Fibres — Total Internal Reflection and Structure

◆ **DEFINITION**

An **optical fibre** is a thin, flexible cylinder of glass or plastic that guides light along its length by repeated total internal reflection.

The **critical angle** θ_c is the angle of incidence, in the denser medium, for which the angle of refraction becomes 90°.

Total internal reflection (TIR) is the complete reflection of light back into the denser medium that occurs when light travelling from a denser to a rarer medium strikes the boundary at an angle greater than the critical angle.

Basic concept

A fibre has two essential parts: a central **core** of refractive index n_1 and a surrounding **cladding** of slightly lower index n_2. Because $n_1 > n_2$, a ray inside the core that meets the boundary at a large enough angle is reflected back into the core instead of escaping. It then meets the opposite wall, is reflected again, and so on. Light is trapped and travels down the fibre in a long zig-zag.

Two points are worth pausing on. First, the cladding is *not* just a protective coat — without it there would be no index step and no guiding at all. Second, TIR is a *perfect* reflection: unlike a metal mirror, which absorbs a percentage at every bounce, TIR loses nothing. That is why light can travel tens of kilometres in a fibre.

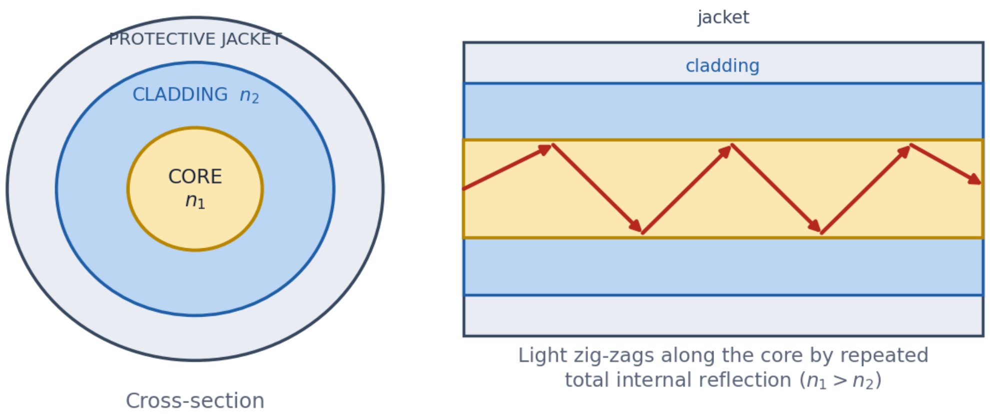


Figure 1.9 — Structure of an optical fibre, in cross-section and along its length.

Typical dimensions: core 8–10 μm (single mode) or 50–62.5 μm (multimode), cladding 125 μm, jacket 250 μm or more. The index difference between core and cladding is only about 0.3–1 %, which is enough because the rays strike the boundary at glancing angles.

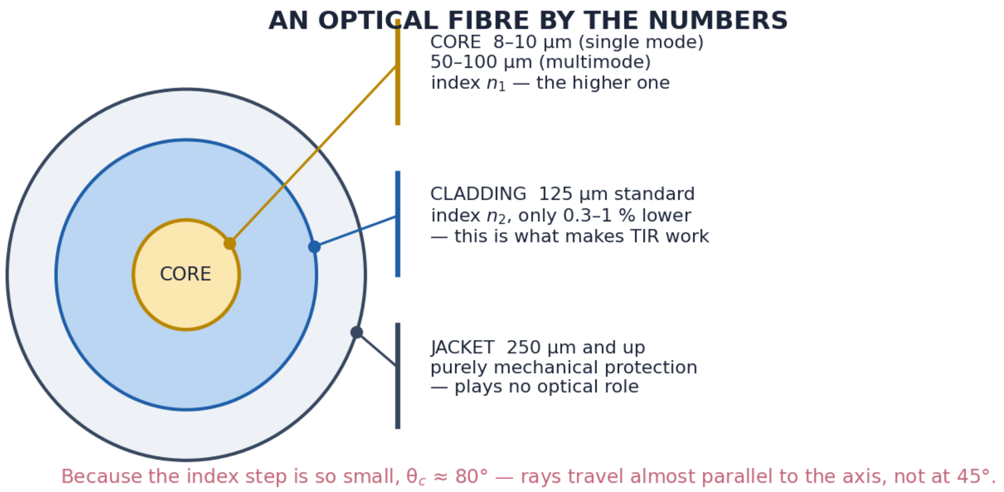


Figure 1.10a — An optical fibre by the numbers.

Conditions for total internal reflection

- Light must travel from the **denser** medium to the **rarer** medium ($n_1 > n_2$).
- The angle of incidence must be **greater than the critical angle** ($\theta_i > \theta_c$).

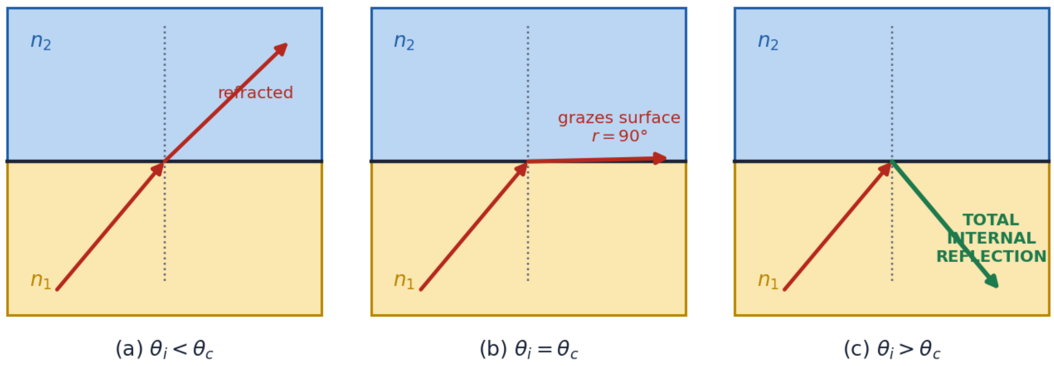


Figure 1.10 — What happens as the angle of incidence is increased.

In (a) most of the light refracts out. In (b), at exactly the critical angle, the refracted ray grazes along the boundary. In (c), beyond the critical angle, refraction is impossible and all the light is reflected back — this is the condition used in a fibre.

Deriving the critical angle

Mathematical derivation — critical angle

● **IMPORTANT NOTE**

Assumptions made

- Light passes from a denser medium of index n_1 into a rarer medium of index n_2, with $n_1 > n_2$.
- Both media are homogeneous and isotropic.
- The boundary is smooth and plane.

1. Apply Snell's law at the boundary.

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

2. At the critical angle, the refracted ray just grazes the surface, so $\theta_r = 90°$ and $\sin \theta_r = 1$.

$$n_1 \sin \theta_c = n_2 \times 1$$

3. Rearrange for $\sin \theta_c$.

$$\sin \theta_c = n_2 / n_1$$

$$\boldsymbol{\theta_c = \sin^{-1} (n_2 / n_1)}$$

critical angle for a core–cladding boundary

Meaning of each symbol

Symbol	What it means	SI unit
θ_c	Critical angle, measured from the normal, inside the core.	degree or rad
n_1	Refractive index of the core (the denser medium).	dimensionless
n_2	Refractive index of the cladding (the rarer medium).	dimensionless

● **IMPORTANT NOTE**

Because n_2/n_1 is only slightly less than 1 in a real fibre, θ_c is large — typically 78–82°. Rays therefore travel almost parallel to the axis and strike the wall at glancing incidence. A common error is to draw the zig-zag at 45°; in a real fibre it is far shallower.

✔ **REMEMBER THESE POINTS**

- Fibre = core (n_1) + cladding (n_2) + protective jacket, with $n_1 > n_2$.
- Guiding is by **total internal reflection**, which is loss-free.
- Two conditions for TIR: denser → rarer, and $\theta_i > \theta_c$.
- $\sin \theta_c = n_2/n_1$.
- Typical $\theta_c \approx 80°$; index difference only 0.3–1 %.

⚠ **COMMON MISTAKES**

- Saying the cladding "protects" the fibre. That is the jacket's job. The cladding provides the refractive-index step.
- Writing $\sin \theta_c = n_1/n_2$ — upside down. The ratio must be less than 1.
- Measuring angles from the surface instead of from the normal.
- Claiming TIR loses a little light at each bounce. It does not — attenuation in a fibre comes from other causes.

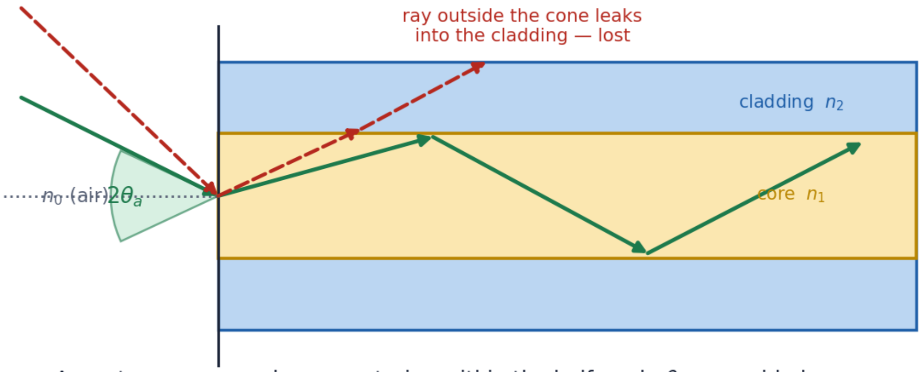
1.9 Acceptance Angle, Acceptance Cone and Numerical Aperture

◆ DEFINITION

The **acceptance angle** θ_a is the largest angle, measured from the fibre axis, at which a ray can enter the core and still undergo total internal reflection — that is, still be guided.

The **acceptance cone** is the cone of half-angle θ_a at the input face of the fibre. Any ray entering within this cone is guided; any ray outside it leaks into the cladding and is lost.

The **numerical aperture (NA)** is the sine of the acceptance angle. It is a pure number that measures the light-gathering ability of the fibre.



Acceptance cone: only rays entering within the half-angle θ_a are guided.

$$NA = \sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

Figure 1.11 — The acceptance angle and the acceptance cone.

The green ray enters within the cone, meets the core–cladding boundary beyond the critical angle, and is guided. The red dashed ray enters at too steep an angle; it meets the boundary at less than the critical angle, refracts into the cladding, and is lost within a few bounces.

Derivation of the numerical aperture

Mathematical derivation — numerical aperture

● IMPORTANT NOTE

Assumptions made

- A ray enters the flat end face of the fibre from a medium of refractive index n_0 (usually air, $n_0 = 1$) at an angle θ_0 to the axis.
- The core has index n_1 and the cladding n_2, with $n_1 > n_2$.
- The end face is perpendicular to the fibre axis.
- We look for the largest θ_0 for which the ray is still guided.

1. Apply Snell's law at the air–core end face.

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

2. Inside the core the ray travels to the core–cladding boundary. From the geometry of the right-angled triangle, the angle it makes with the normal to that boundary is $\varphi = 90° - \theta_1$.

$$\varphi = 90° - \theta_1 \;\rightarrow\; \sin\theta_1 = \cos\varphi$$

3. Substitute this into the first equation.

$$n_0 \sin\theta_0 = n_1 \cos\varphi$$

4. For the ray to be guided we need $\varphi \geq \theta_c$. The limiting case, which gives the largest acceptable θ_0, is $\varphi = \theta_c$. Write $\theta_0 = \theta_a$ for this limiting value.

$$n_0 \sin\theta_a = n_1 \cos\theta_c$$

5. Now use $\cos\theta_c = \sqrt{(1 - \sin^2\theta_c)}$ together with $\sin\theta_c = n_2/n_1$ from the previous topic.

$$\cos\theta_c = \sqrt{(1 - n_2^2/n_1^2)} = \sqrt{(n_1^2 - n_2^2)} / n_1$$

6. Substitute back.

$$n_0 \sin\theta_a = n_1 \cdot \sqrt{(n_1^2 - n_2^2)} / n_1 = \sqrt{(n_1^2 - n_2^2)}$$

7. Finally put $n_0 = 1$ for air.

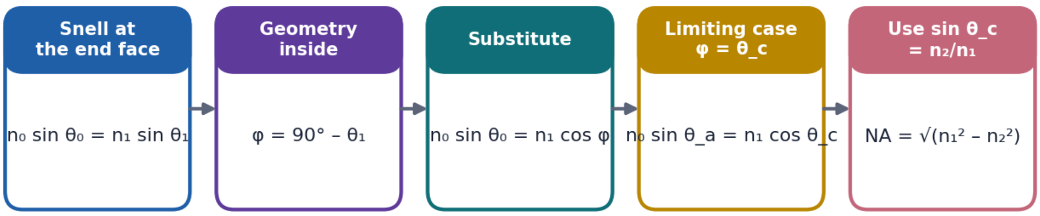
$$\mathbf{NA = \sin\theta_a = \sqrt{(\,n_1^2 - n_2^2\,)}}$$

numerical aperture of a step-index fibre in air

Meaning of each symbol

Symbol	What it means	SI unit
NA	Numerical aperture — the light-gathering power of the fibre.	dimensionless
θ_a	Acceptance angle (half-angle of the acceptance cone).	degree or rad
n_0	Refractive index of the medium outside the fibre (1 for air).	dimensionless
n_1	Refractive index of the core.	dimensionless
n_2	Refractive index of the cladding.	dimensionless

THE NA DERIVATION IN FIVE MOVES



Learn the five moves, not the final line — examiners give most of the marks for the middle three.

Figure 1.12a — The numerical-aperture derivation in five moves.

Fractional index change and the alternative form

The **fractional refractive-index change** (or relative index difference) is defined as

$$\Delta = (n_1 - n_2) / n_1$$

Since $n_1^2 - n_2^2 = (n_1 + n_2)(n_1 - n_2)$, and since $n_1 \approx n_2$ in a real fibre so that $n_1 + n_2 \approx 2n_1$, we get the very useful approximation

$$NA \approx n_1 \sqrt{2\Delta}$$

valid when $\Delta \ll 1$, which is always true for practical fibres

IMPORTANT NOTE

The engineer's dilemma. A large NA gathers more light from the source, which sounds good. But a large NA also means rays enter over a wide range of angles, so they take paths of very different lengths and arrive at different times — the pulse spreads out. This is *intermodal dispersion*, and it limits how fast you can send data. So NA is always a compromise: large for short-distance illumination and sensing, small for long-distance high-speed communication. Typical values are 0.2–0.5 for multimode fibre and 0.1–0.14 for single mode.

⚙ REAL-LIFE APPLICATIONS

- NA determines which source can be used: an LED has a wide emission cone and needs a large-NA multimode fibre; a laser diode has a narrow cone and suits a small-NA single-mode fibre.
- In fibre-optic endoscopes and illuminators a large NA is chosen deliberately to collect as much light as possible.
- NA is the number quoted on every commercial fibre datasheet, and it decides which connectors and splices are compatible.

✔ REMEMBER THESE POINTS

- $NA = \sin\theta_a = \sqrt{n_1^2 - n_2^2}$ — memorise this.
- Also $NA \approx n_1\sqrt{2\Delta}$, with $\Delta = (n_1 - n_2)/n_1$.
- The acceptance cone has half-angle θ_a; its full angle is $2\theta_a$.
- NA is dimensionless and always less than 1 for a fibre in air.
- Large NA → more light collected but more dispersion.

⚠ COMMON MISTAKES

- Confusing the acceptance angle (outside the fibre, from the axis) with the critical angle (inside the fibre, from the normal to the wall). They are complementary, not equal.
- Writing $NA = \sqrt{n_1 - n_2}$. The squares matter.
- Reporting $2\theta_a$ when the question asks for θ_a, or vice versa. Read the question carefully.
- Giving NA a unit. It has none.
- Forgetting the n_0 in the denominator when the fibre is used in water or in a fluid, where $n_0 \neq 1$.

★ **EXAM TIP**

- "Derive the expression for the numerical aperture of an optical fibre" is one of the most reliably asked derivations in the whole syllabus. Learn every step.
- Always draw Figure 1.11 with the derivation — the geometry step $\varphi = 90° - \theta_1$ is impossible to justify without it.
- State the assumption $n_0 = 1$ explicitly; it is worth a mark.

1.10 Classification of Optical Fibres

Fibres are classified in two independent ways: by the **number of modes** they support, and by the **shape of the refractive-index profile** of the core. Combining the two gives the three types you actually meet in practice.

◆ **DEFINITION**

A **mode** is one allowed pattern of the electromagnetic field that can propagate along the fibre without changing its transverse shape. Loosely, you can think of each mode as one allowed zig-zag path.

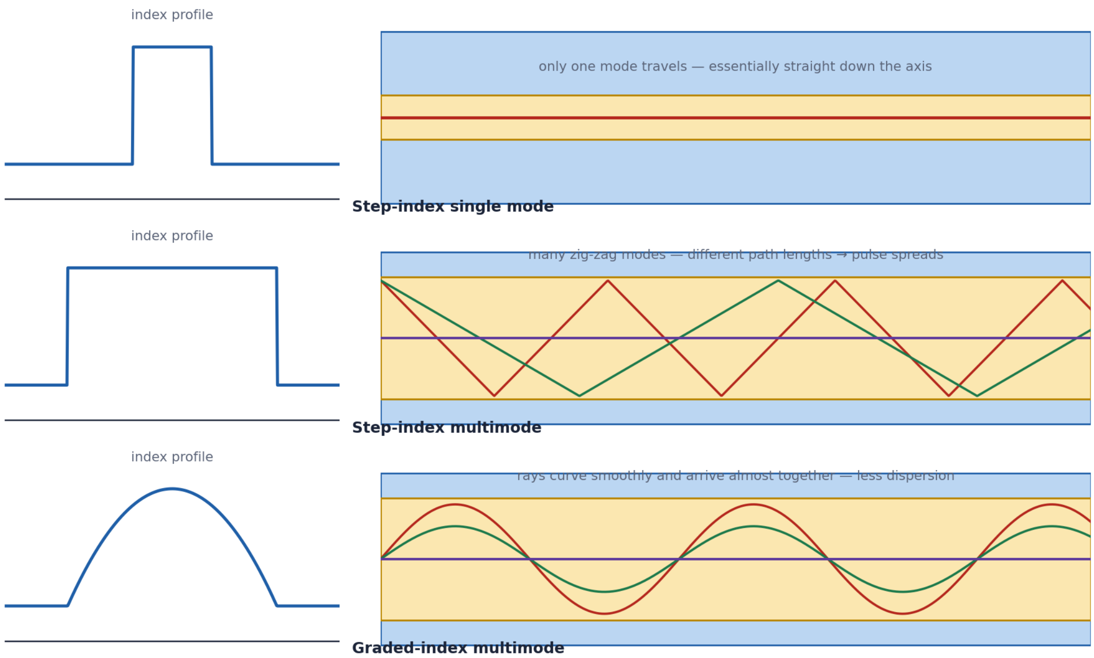


Figure 1.12 — The three practical fibre types: index profile on the left, ray path on the right.

In step-index fibre the index changes abruptly at the boundary and rays travel in straight lines between reflections. In graded-index fibre the index falls smoothly from the axis outwards, so rays are continuously refracted and follow smooth curves — an off-axis ray travels further but spends more of its journey in the lower-index outer region where light moves faster, so all rays arrive at nearly the same time.

A. Classification by number of modes

Feature	Single-mode fibre (SMF)	Multimode fibre (MMF)
Core diameter	8–10 µm	50–100 µm
Number of modes	One (the fundamental mode)	Hundreds to thousands
Numerical aperture	Small, ≈ 0.1–0.14	Larger, ≈ 0.2–0.5
Source used	Laser diode	LED or laser diode
Intermodal dispersion	None (only one mode)	Significant
Bandwidth	Very high (tens of GHz·km)	Lower (tens to hundreds of MHz·km)
Distance	Tens to hundreds of km	Up to about 2 km
Splicing and coupling	Difficult — needs precise alignment	Easier
Cost of fibre / of system	Fibre cheaper, system dearer	Fibre dearer, system cheaper
Typical use	Long-haul telecom, undersea cables, FTTH	LAN, data centres, short links, sensors

Table 1.7 — Single-mode versus multimode fibre.

The number of modes a fibre supports is governed by the **V-number** (normalised frequency):

$$V = (2\pi a/\lambda) \times NA = (2\pi a/\lambda)\sqrt{n_1^2 - n_2^2}$$

Meaning of each symbol

Symbol	What it means	SI unit
V	V-number or normalised frequency — decides how many modes propagate.	dimensionless
a	Radius of the core.	m
λ	Wavelength of the light in free space.	m
NA	Numerical aperture of the fibre.	dimensionless

The fibre carries only a single mode when $V < 2.405$ (the first zero of the Bessel function J_0). For larger V the approximate number of modes is $N \approx V^2/2$ for a step-index fibre, and $N \approx V^2/4$ for a graded-index fibre.

B. Classification by index profile

Feature	Step-index fibre	Graded-index fibre
Core index	Uniform; changes abruptly at the cladding	Highest on the axis, falling smoothly outwards
Index profile	$n(r) = n_1$ for $r < a$; n_2 for $r > a$	$n(r) = n_1\sqrt{[1 - 2\Delta(r/a)^{\alpha}]}$, usually with $\alpha \approx 2$ (parabolic)
Ray path	Straight lines with sharp reflections (zig-zag)	Smooth sinusoidal curves

Feature	Step-index fibre	Graded-index fibre
Intermodal dispersion	Large	Much smaller — rays are equalised
Bandwidth	Lower	Higher
Manufacture	Simple, cheaper	Complex, costlier
Typical use	Short links, illumination, sensors	Medium-distance LAN and campus backbones

Table 1.8 — Step-index versus graded-index fibre.

✓ REMEMBER THESE POINTS

- Two classifications: by modes (single/multi) and by profile (step/graded).
- $V = (2\pi a/\lambda)\cdot NA$; single mode when $V < 2.405$.
- Number of modes $\approx V^2/2$ (step) and $\approx V^2/4$ (graded).
- Single mode → no intermodal dispersion → long distance, high bandwidth.
- Graded index reduces intermodal dispersion by equalising transit times.

⚠ COMMON MISTAKES

- Using the core *diameter* in place of the radius a in the V-number formula. This halves your answer.
- Saying single-mode fibre has “no dispersion”. It has no *intermodal* dispersion; material and waveguide dispersion remain.
- Confusing the two classifications — “single mode” and “step index” are not opposites; most single-mode fibre is step index.

1.11 Attenuation in Optical Fibres

◆ DEFINITION

Attenuation is the loss of optical power as a signal travels along a fibre.

The **attenuation coefficient** α is the loss expressed in decibels per kilometre of fibre.

$$\alpha = (10 / L) \cdot \log_{10} (P_{in} / P_{out}) \quad \text{dB/km}$$

Meaning of each symbol

Symbol	What it means	SI unit
α	Attenuation coefficient — loss per unit length.	dB km^{-1}
L	Length of the fibre.	km
P_{in}	Optical power launched into the fibre.	W (or mW)
P_{out}	Optical power emerging at the far end.	W (or mW)

IMPORTANT NOTE

Watch the sign. Written as above, with P_{in} on top, α comes out positive. Many textbooks write $\alpha = -(10/L)\log(P_{out}/P_{in})$, which is exactly the same thing. Whichever form you use, an attenuation coefficient is quoted as a positive number of dB/km, and the total loss in decibels is simply α × L.

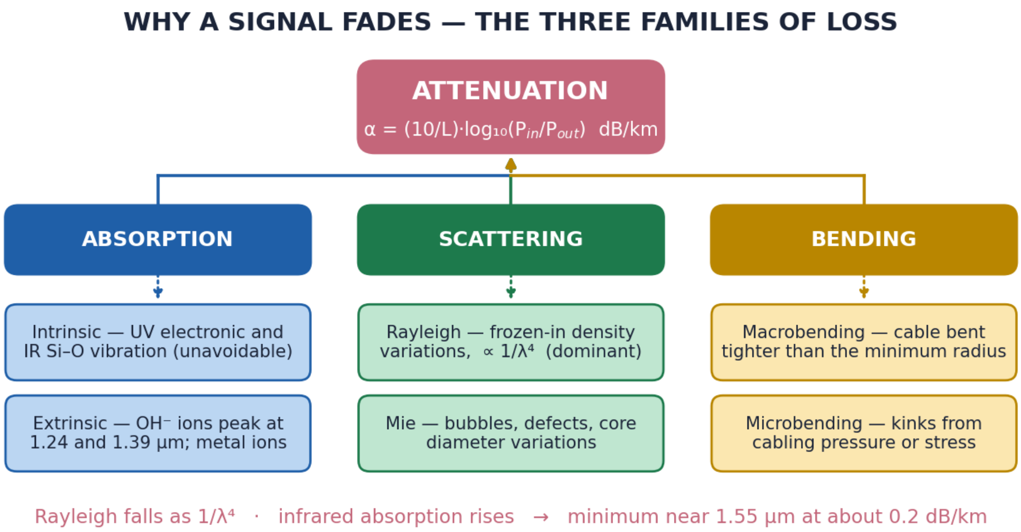


Figure 1.14 — The three families of loss in an optical fibre.

Causes of attenuation

The syllabus asks for the causes, so learn them under three headings.

1. Absorption losses

- **Intrinsic absorption** — the glass itself absorbs. In the ultraviolet this is electronic absorption; in the infrared it is molecular vibration of the Si–O bond. Both are unavoidable properties of silica and they set the floor on how good a fibre can ever be.
- **Extrinsic absorption** — caused by impurities. The worst offender is the hydroxyl (OH^-) ion left over from water in manufacture, which produces sharp absorption peaks near 1.24 µm and 1.39 µm. Transition metal ions such as iron, copper and chromium also absorb. Modern "low-water-peak" fibre has almost eliminated the OH^- peaks.

2. Scattering losses

- **Rayleigh scattering** — the dominant loss in modern fibre. Tiny random variations of density and composition, frozen in when the glass solidified, scatter light in all directions. The loss varies as $\mathbf{1/\lambda^4}$, so it falls very steeply with increasing wavelength. This single fact is why long-haul systems moved from 850 nm to 1550 nm.
- **Mie scattering** — caused by imperfections comparable in size to the wavelength: bubbles, defects, variations in core diameter. Good manufacture keeps this small.

3. Bending losses

- **Macrobending** — a large-radius bend in the cable. At the bend the angle of incidence at the wall falls below the critical angle and light escapes. Every fibre has a specified minimum bend radius.

- **Microbending** — microscopic kinks caused by uneven pressure during cabling or by temperature-induced stress. They couple light out of the guided mode.

Two further losses are usually grouped with these in practice:

- **Coupling and connector losses** at joints, splices and source–fibre interfaces.
- **Dispersion**, which does not remove power but spreads the pulse and so limits the usable data rate.

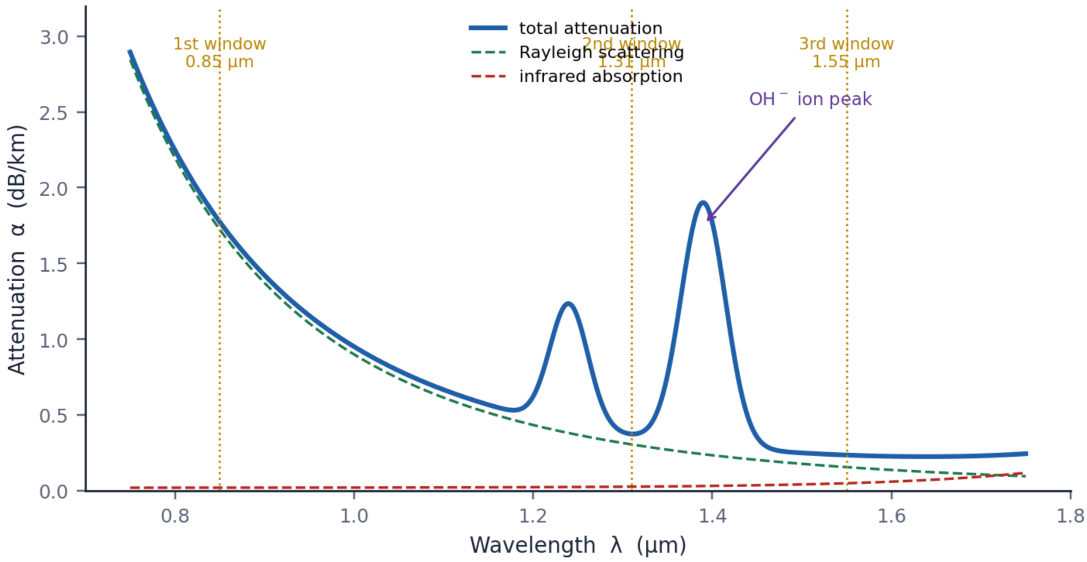


Figure 1.13 — Attenuation of a silica fibre against wavelength.

Read the graph as a competition. On the left, Rayleigh scattering ($\propto 1/\lambda^4$) falls steeply. On the right, infrared absorption rises steeply. Between them lies a minimum near 1.55 µm, where the total loss is about 0.2 dB/km — the lowest loss achievable in silica, and the reason 1.55 µm is the standard wavelength for long-haul systems. The sharp peaks are due to OH^- ions. The three marked transmission windows are 0.85 µm, 1.31 µm (zero dispersion) and 1.55 µm (minimum loss).

Window	Wavelength	Typical loss	Why it is used
First	0.85 µm	≈ 2–3 dB/km	Cheap GaAs sources and silicon detectors; short links
Second	1.31 µm	≈ 0.35 dB/km	Material dispersion in silica is zero here
Third	1.55 µm	≈ 0.2 dB/km	Lowest attenuation; erbium-doped amplifiers work here

Table 1.9 — The three optical transmission windows.

✔ REMEMBER THESE POINTS

- $\alpha = (10/L)\cdot\log_{10}(P_{in}/P_{out})$ dB/km.
- Three causes: absorption (intrinsic + extrinsic), scattering (Rayleigh + Mie), bending (macro + micro).
- Rayleigh scattering $\propto 1/\lambda^4$ — the dominant loss.
- OH^- ions cause peaks at 1.24 µm and 1.39 µm.
- Minimum loss ≈ 0.2 dB/km at 1.55 µm.
- Windows: 0.85, 1.31 (zero dispersion), 1.55 µm (minimum loss).

⚠ COMMON MISTAKES

- Using natural logarithm instead of log to base 10. The decibel is defined with $\log_{10}$.
- Forgetting to convert the length to kilometres — the answer must be per kilometre.
- Writing Rayleigh scattering as $\propto 1/\lambda^2$ or $1/\lambda$. It is $\mathbf{1/\lambda^4}$.
- Listing dispersion as a cause of attenuation. Dispersion spreads the pulse; it does not by itself remove power.

1.12 Optical Fibre Communication — Advantages

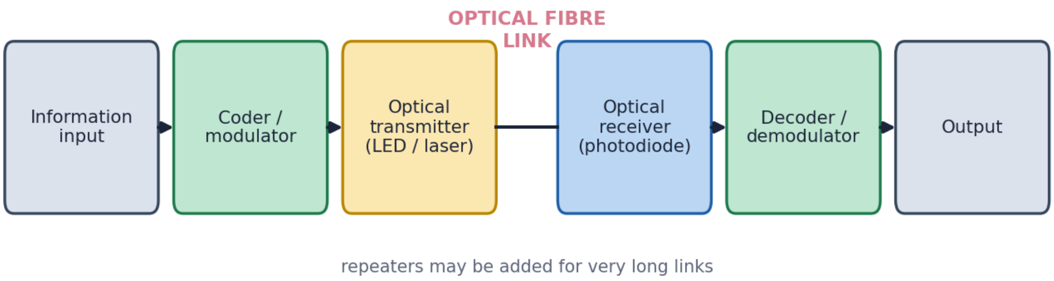


Figure 1.14 — Block diagram of an optical fibre communication system.

The electrical input is coded and used to modulate an optical source (LED or laser diode). The light travels down the fibre, is detected by a photodiode (PIN or avalanche), and is amplified, decoded and delivered. On long links, optical amplifiers or regenerators are inserted at intervals.

Advantages over copper cable

Advantage	Why it follows from the physics
Enormous bandwidth	The optical carrier frequency is about 10^5 times higher than a microwave carrier, and the information capacity scales with carrier frequency. A single fibre can carry terabits per second.
Very low loss	About 0.2 dB/km against 10–20 dB/km for coaxial cable — so repeaters can be 80–100 km apart instead of every 1–2 km.

Advantage	Why it follows from the physics
Complete immunity to electromagnetic interference	Light is not affected by electric or magnetic fields, so fibre works beside power lines, motors and welding equipment without pick-up, and it produces no interference of its own.
Excellent security	There is no radiated field to tap, and any attempt to tap the fibre causes a detectable loss of power.
Small size and light weight	A fibre is about the thickness of a human hair. A fibre cable of the same capacity weighs a small fraction of a copper cable — decisive in aircraft, ships and crowded ducts.
Electrical isolation and safety	Glass is an insulator, so there is no short-circuit risk, no earth loop and no spark hazard. Fibre can be used safely in explosive and chemical environments.
No crosstalk	Light stays inside its own fibre, so signals in adjacent fibres do not interfere.
Abundant, cheap raw material	Silica is made from sand, whereas copper is a limited and expensive resource.
Long life and reliability	Glass does not corrode and is not affected by moisture or temperature in the way that copper is.

Table 1.10 — Advantages of optical fibre communication.

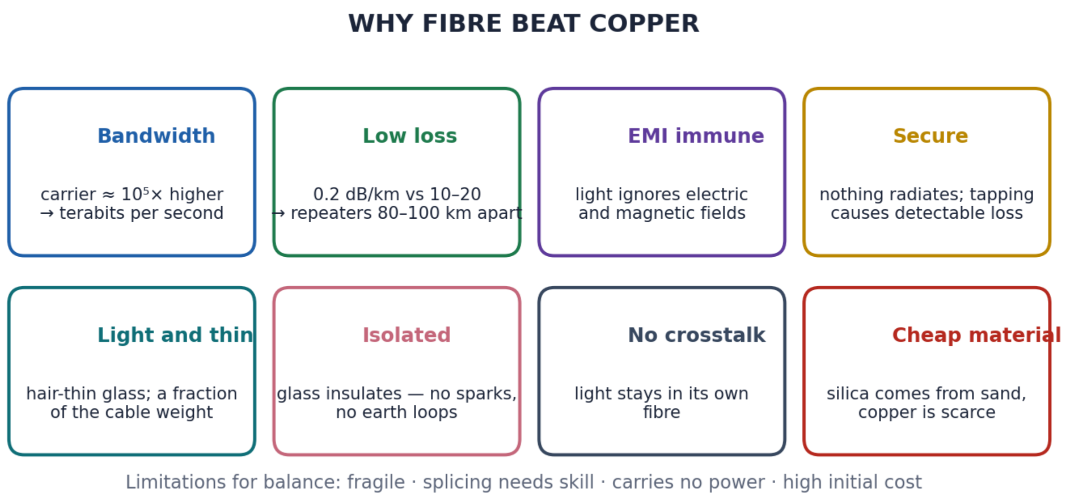


Figure 1.16 — Why fibre replaced copper.

Limitations, for balance

- Fibre is fragile and can break if bent too sharply.
- Splicing and connecting need skilled technicians and costly equipment.
- Cannot carry electrical power to remote equipment, so a separate power feed is needed.
- Higher initial installation cost, although the cost per bit is now far lower than copper.

⚙ REAL-LIFE APPLICATIONS

- Long-haul and undersea telecommunication backbones.
- Fibre-to-the-home broadband and cable television.
- Data-centre and campus local area networks.
- Medical endoscopes and surgical illumination.
- Fibre-optic sensors for temperature, strain, pressure and chemical detection in civil and aerospace structures.
- Military and avionic data buses, where immunity to interference is critical.

✔ REMEMBER THESE POINTS

- Blocks: input → coder → optical transmitter → fibre → optical receiver → decoder → output.
- Main advantages: huge bandwidth, very low loss, EMI immunity, security, light weight, electrical isolation, no crosstalk.
- Repeater spacing 80–100 km against 1–2 km for coaxial cable.
- Limitations: fragile, difficult splicing, no power delivery, high initial cost.

★ EXAM TIP

- Draw the block diagram whenever advantages are asked — it frames the answer and often carries marks of its own.
- Give at least six advantages, each with its one-line reason. A bare list scores poorly.
- Adding two or three limitations shows balance and is often explicitly asked for.

↻ QUICK REVISION BOX — Optical Fibres

The four formulas

$\sin\theta_c = n_2/n_1$ · $NA = \sin\theta_a = \sqrt{n_1^2 - n_2^2}$ · $V = (2\pi a/\lambda)\cdot NA$ · $\alpha = (10/L)\log_{10}(P_{in}/P_{out})$

One-line definitions

- **Critical angle** — angle of incidence for which refraction is at 90°.
- **Acceptance angle** — largest entry angle from the axis for which a ray is still guided.
- **Numerical aperture** — the sine of the acceptance angle; the light-gathering power.
- **Mode** — one allowed field pattern that propagates unchanged.
- **Attenuation coefficient** — loss in dB per km.

Numbers to remember

- Single mode: core 8–10 μm, NA ≈ 0.1–0.14, V < 2.405.
- Multimode: core 50–100 μm, NA ≈ 0.2–0.5.
- Cladding diameter 125 μm (an industry standard).
- Minimum loss 0.2 dB/km at 1.55 μm.
- Number of modes ≈ $V^2/2$ (step index).

Cause-and-effect chains worth reciting

- $n_1 > n_2$ → TIR possible → light guided.
- Larger NA → more light in → more intermodal dispersion → lower bandwidth.
- Longer λ → less Rayleigh scattering ($1/\lambda^4$) → lower loss, until infrared absorption takes over.
- Graded index → rays equalised → less dispersion → higher bandwidth.

Unit I — Formula Sheet

Every formula that appears in this unit, in one place. Learn the equation **and** the meaning of each symbol — SPPU examiners regularly ask for the meaning of terms as a separate part of a question.

Quantity / law	Equation	Symbols and SI units	When you use it
Photon energy	$E = h\nu = hc/\lambda$	E in J; $h = 6.626\times10^{-34}$ J s; ν in Hz; λ in m; $c = 3\times10^{8}$ m s^{-1}	Finding the energy of the laser transition, or λ from the level gap.
Boltzmann population ratio	$N_2/N_1 = e^{-\Delta E/k_BT}$	ΔE in J; $k_B = 1.38\times10^{-23}$ J K^{-1}; T in K	Showing that inversion cannot occur at equilibrium.
Einstein relation	$A_{21}/B_{21} = 8\pi h\nu^3/c^3$	A_{21} in s^{-1}; B_{21} in $m^3\ J^{-1}\ s^{-2}$	Comparing spontaneous with stimulated emission at a given frequency.
Cavity mode condition	$L = n\lambda/2,\ \ \nu_n = nc/2L$	L in m; n an integer; λ in m; ν in Hz	Finding the allowed longitudinal modes of a cavity.
Mode spacing	$\Delta\nu = c/2L$	Δν in Hz; L in m	Finding the frequency gap between adjacent modes.
Beam divergence	$\theta = (d_2 - d_1)/(x_2 - x_1)$	θ in rad; d and x in m	Numericals on how much a laser beam spreads.
Coherence length	$L_c = c\,\tau_c = \lambda^2/\Delta\lambda$	L_c in m; τ_c in s; λ, Δλ in m	Judging whether a source is coherent enough for holography.
Critical angle	$\theta_c = \sin^{-1}(n_2/n_1)$	θ_c in degree; n_1, n_2 dimensionless	Finding the minimum angle for total internal reflection.
Numerical aperture	$NA = \sin\theta_a = \sqrt{(n_1^2 - n_2^2)}$	NA and n dimensionless; θ_a in degree	The most frequently used fibre formula — light-gathering power.
NA in terms of Δ	$NA \approx n_1\sqrt{(2\Delta)},\ \Delta = (n_1 - n_2)/n_1$	Δ dimensionless (often quoted as a percentage)	When the fractional index change is given instead of n_2.
Acceptance angle	$\theta_a = \sin^{-1}(NA/n_0)$	$n_0 = 1$ for air	Converting NA to the acceptance angle; full cone = $2\theta_a$.
V-number	$V = (2\pi a/\lambda)\sqrt{(n_1^2 - n_2^2)}$	a = core **radius** in m; λ in m; V dimensionless	Deciding whether a fibre is single mode ($V < 2.405$).
Number of modes	$N \approx V^2/2$ (step), $N \approx V^2/4$ (graded)	N dimensionless	Estimating how many modes a fibre carries.

Quantity / law	Equation	Symbols and SI units	When you use it
Attenuation coefficient	$\alpha = (10/L)\log_{10}(P_{in}/P_{out})$	α in dB km^{-1}; L in km; P in W	Finding the loss of a fibre link, or the output power after L km.
Output power after L km	$P_{out} = P_{in} \times 10^{-\alpha L/10}$	P in W or mW; α in dB/km; L in km	The reverse calculation — how much power survives.

Unit I — Numerical Problems

Problems are graded: the first few are direct substitution, the later ones need two or three steps or a rearrangement. Work each one on paper before reading the solution.

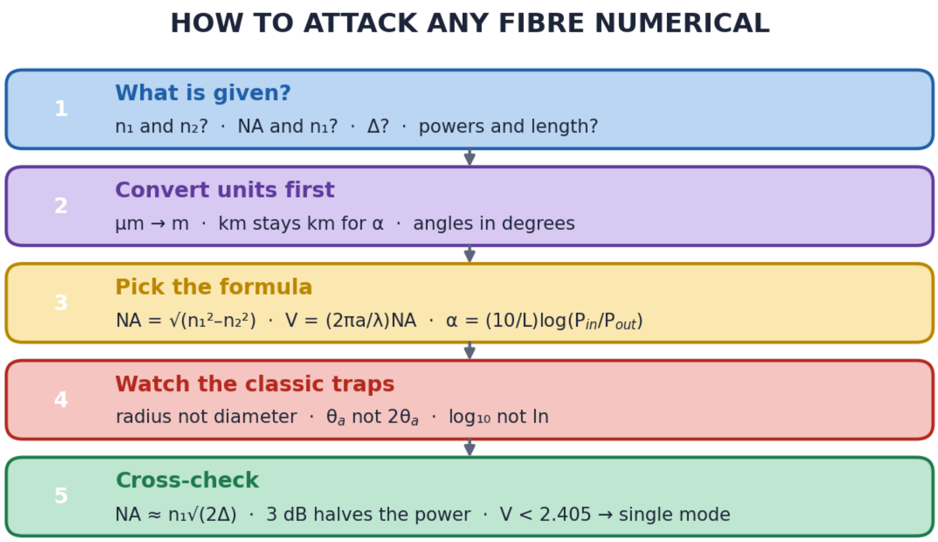


Figure 1.17 — A reliable routine for any fibre numerical.

➤ Problem 1

A He–Ne laser emits light of wavelength 632.8 nm. Calculate the energy of one photon in joule and in electronvolt.

Given data

- $\lambda = 632.8$ nm $= 632.8 \times 10^{-9}$ m
- $h = 6.626 \times 10^{-34}$ J s
- $c = 3 \times 10^{8}$ m s^{-1}

Formula used

$$E = hc / \lambda$$

Step-by-step solution

1. Substitute: $E = (6.626 \times 10^{-34} \times 3 \times 10^{8}) / (632.8 \times 10^{-9})$.
2. Numerator: $6.626 \times 3 = 19.878$, so 19.878×10^{-26} J m.
3. Divide: $E = 19.878 \times 10^{-26} / 632.8 \times 10^{-9} = 3.141 \times 10^{-19}$ J.
4. Convert to eV by dividing by 1.6×10^{-19} J/eV: $E = 3.141 \times 10^{-19} / 1.6 \times 10^{-19} = 1.963$ eV.

Final answer

$$E = 3.14 \times 10^{-19}\ J \approx 1.96\ eV$$

Exam tip

Keep the shortcut E(eV) = 1240 / λ(nm) in your head. Here 1240/632.8 = 1.96 eV — it checks the answer in two seconds.

➤ Problem 2

The two mirrors of a laser cavity are 40 cm apart. Find the frequency spacing between adjacent longitudinal modes, and the number of half-wavelengths of 632.8 nm light that fit between the mirrors.

Given data

- L = 40 cm = 0.40 m
- $\lambda = 632.8 \times 10^{-9}$ m
- $c = 3 \times 10^{8}\ m\ s^{-1}$

Formula used

$$\Delta\nu = c / 2L$$

$$L = n\,\lambda / 2$$

Step-by-step solution

1. Mode spacing: $\Delta\nu = 3 \times 10^{8} / (2 \times 0.40) = 3 \times 10^{8} / 0.80 = 3.75 \times 10^{8}$ Hz.
2. That is 375 MHz.
3. Mode number: $n = 2L / \lambda = (2 \times 0.40) / (632.8 \times 10^{-9})$.
4. $n = 0.80 / 632.8 \times 10^{-9} = 1.264 \times 10^{6}$.

Final answer

$$\Delta\nu = 375\ MHz;\ n \approx 1.26 \times 10^{6}\ \text{half-wavelengths}$$

Exam tip

The huge mode number is the point of the question — it shows why the spacing between modes is tiny compared with the optical frequency, and hence why a laser line is so narrow.

➤ Problem 3

A laser beam of initial diameter 3 mm has a diameter of 8 mm after travelling 5 m. Calculate the divergence in milliradian, and estimate the beam diameter at 1 km.

Given data

- $d_1 = 3\ mm = 3 \times 10^{-3}$ m
- $d_2 = 8\ mm = 8 \times 10^{-3}$ m
- $x_2 - x_1 = 5$ m

Formula used

$$\theta = (d_2 - d_1) / (x_2 - x_1)$$

Step-by-step solution

1. Change in diameter: 8 – 3 = 5 mm = 5×10^{-3} m.
2. $\theta = 5 \times 10^{-3} / 5 = 1 \times 10^{-3}$ rad = 1 mrad.
3. At 1 km the extra spread is $\theta \times 1000 = 1 \times 10^{-3} \times 1000 = 1$ m.
4. Beam diameter at 1 km ≈ 3 mm + 1 m ≈ 1.003 m ≈ 1 m.

Final answer

$$\theta = 1 \text{ mrad; beam diameter at 1 km} \approx 1 \text{ m}$$

Exam tip

Note how the initial 3 mm becomes negligible over a kilometre. Examiners like this because it gives a physical feel for how "small" 1 mrad really is.

➤ Problem 4

An optical fibre has a core of refractive index 1.50 and a cladding of refractive index 1.47. Calculate (i) the critical angle, (ii) the numerical aperture, (iii) the acceptance angle, and (iv) the fractional index change.

Given data

- $n_1 = 1.50$
- $n_2 = 1.47$
- $n_0 = 1$ (air)

Formula used

$$\sin\theta_c = n_2/n_1$$
$$NA = \sqrt{(n_1^2 - n_2^2)}$$
$$\theta_a = \sin^{-1}(NA)$$
$$\Delta = (n_1 - n_2)/n_1$$

Step-by-step solution

1. (i) $\sin\theta_c = 1.47 / 1.50 = 0.98$, so $\theta_c = \sin^{-1}(0.98) = 78.52°$.
2. (ii) $n_1^2 = 2.2500$ and $n_2^2 = 2.1609$, so $n_1^2 - n_2^2 = 0.0891$.
3. $NA = \sqrt{0.0891} = 0.2985 \approx 0.30$.
4. (iii) $\theta_a = \sin^{-1}(0.2985) = 17.37°$. The full acceptance cone is $2\theta_a = 34.74°$.
5. (iv) $\Delta = (1.50 - 1.47)/1.50 = 0.03/1.50 = 0.02$, i.e. 2 %.
6. Check with the approximation: $NA \approx n_1\sqrt{(2\Delta)} = 1.50 \times \sqrt{0.04} = 1.50 \times 0.20 = 0.30$ — it agrees.

Final answer

$$\theta_c = 78.5°, NA = 0.30, \theta_a = 17.4° \text{ (cone } 34.7°\text{)}, \Delta = 2\ \%$$

Exam tip

This is the archetypal fibre numerical and appears in some form almost every year. Always finish by cross-checking NA with $n_1\sqrt{(2\Delta)}$ — it costs ten seconds and catches arithmetic slips.

➤ Problem 5

A step-index fibre has a core radius of 25 μm, a core index of 1.48 and a cladding index of 1.46. Light of wavelength 1.30 μm is used. Find the V-number and the number of modes the fibre supports. Is it single mode?

Given data

- $a = 25\ \mu m = 25 \times 10^{-6}$ m
- $n_1 = 1.48$
- $n_2 = 1.46$
- $\lambda = 1.30\ \mu m = 1.30 \times 10^{-6}$ m

Formula used

$$NA = \sqrt{(n_1^2 - n_2^2)}$$

$$V = (2\pi a/\lambda) \times NA$$

$$N \approx V^2/2$$

Step-by-step solution

1. NA: $n_1^2 = 2.1904$, $n_2^2 = 2.1316$, difference = 0.0588.
2. $NA = \sqrt{0.0588} = 0.2425$.
3. $V = (2\pi \times 25 \times 10^{-6} / 1.30 \times 10^{-6}) \times 0.2425$.
4. The bracket: $2\pi \times 25 / 1.30 = 157.08 / 1.30 = 120.83$.
5. $V = 120.83 \times 0.2425 = 29.30$.
6. Number of modes: $N \approx V^2/2 = (29.30)^2/2 = 858.5/2 = 429$.
7. Since V = 29.3 is much greater than 2.405, the fibre is **multimode**.

Final answer

NA = 0.243, V = 29.3, N ≈ 429 modes — multimode

Exam tip

The commonest error here is using the diameter instead of the radius. If your V comes out roughly double what you expect, check that first.

➤ Problem 6

An optical fibre 15 km long has an attenuation coefficient of 0.4 dB/km. If 2 mW of optical power is launched into it, what power emerges at the far end?

Given data

- L = 15 km
- $\alpha = 0.4$ dB/km
- $P_{in} = 2$ mW

Formula used

$$\text{Total loss (dB)} = \alpha \times L$$

$$P_{out} = P_{in} \times 10^{-\alpha L/10}$$

Step-by-step solution

1. Total loss = 0.4 × 15 = 6 dB.
2. $P_{out} = 2 \times 10^{-6/10} = 2 \times 10^{-0.6}$ mW.
3. $10^{-0.6} = 0.2512$.
4. $P_{out} = 2 \times 0.2512 = 0.5024$ mW.

Final answer

$$\mathbf{P_{out} \approx 0.50\ mW}$$

Exam tip

A 3 dB loss always halves the power and a 10 dB loss always divides it by ten. Here 6 dB ≈ two 3 dB steps, so the power should fall to about a quarter — 0.5 mW from 2 mW. Use this to sanity-check every attenuation answer.

➤ Problem 7

The input power to a fibre is 100 μW and the output power after 8 km is 25 μW. Calculate the attenuation coefficient of the fibre.

Given data

- P_{in} = 100 µW
- P_{out} = 25 µW
- L = 8 km

Formula used

$$\alpha = (10/L)\ \log_{10}(P_{in}/P_{out})$$

Step-by-step solution

1. Power ratio: P_{in}/P_{out} = 100/25 = 4.
2. $\log_{10}(4) = 0.6021$.
3. α = (10/8) × 0.6021 = 1.25 × 0.6021 = 0.7526 dB/km.

Final answer

$$\boldsymbol{\alpha \approx 0.75\ \text{dB/km}}$$

Exam tip

The units of the powers cancel in the ratio, so there is no need to convert µW to W. Recognising that saves time and avoids errors.

➤ Problem 8

A ruby laser emits a pulse of energy 2 J lasting 5 ns at a wavelength of 694.3 nm. Calculate (i) the peak power of the pulse and (ii) the number of photons emitted.

Given data

- E_{pulse} = 2 J
- $t = 5\ \text{ns} = 5 \times 10^{-9}$ s
- $\lambda = 694.3\ \text{nm} = 694.3 \times 10^{-9}$ m

Formula used

$$P = E_{pulse} / t$$

$$E_{photon} = hc/\lambda$$

$$N = E_{pulse} / E_{photon}$$

Step-by-step solution

1. (i) $P = 2 / (5 \times 10^{-9}) = 0.4 \times 10^{9} = 4 \times 10^{8}$ W = 400 MW.
2. (ii) Energy of one photon: $E = (6.626 \times 10^{-34} \times 3 \times 10^{8}) / (694.3 \times 10^{-9})$.
3. $E = 19.878 \times 10^{-26} / 694.3 \times 10^{-9} = 2.863 \times 10^{-19}$ J.
4. $N = 2 / (2.863 \times 10^{-19}) = 6.99 \times 10^{18}$ photons.

Final answer

$$\textbf{Peak power = 400 MW; N} \approx \mathbf{7 \times 10^{18}}\ \textbf{photons}$$

Exam tip

400 MW from a 2 J pulse looks impossible until you notice how short 5 ns is. This is exactly the point of the question — a laser's power comes from concentrating modest energy into a very short time.

➤ Problem 9

Calculate the ratio of the population of two energy levels separated by 2.0 eV at a temperature of 300 K. Comment on the result.

Given data

- $\Delta E = 2.0\ \text{eV} = 2.0 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ J
- $T = 300$ K
- $k_B = 1.38 \times 10^{-23}$ J K^{-1}

Formula used

$$N_2/N_1 = \exp(-\Delta E / k_B T)$$

Step-by-step solution

1. $k_B T = 1.38 \times 10^{-23} \times 300 = 4.14 \times 10^{-21}$ J.
2. $\Delta E / k_B T = 3.2 \times 10^{-19} / 4.14 \times 10^{-21} = 77.3$.
3. $N_2/N_1 = e^{-77.3} \approx 2.6 \times 10^{-34}$.

Final answer

$$\mathbf{N_2/N_1 \approx 2.6 \times 10^{-34}} \text{ — essentially zero}$$

Exam tip

Always add the comment the question asks for: at equilibrium the upper level is empty for all practical purposes, so population inversion can never occur naturally and must be forced by pumping. The comment is usually worth a mark.

➤ Problem 10

A fibre has a numerical aperture of 0.22 and a core refractive index of 1.47. Find the refractive index of the cladding and the acceptance angle when the fibre is used (i) in air and (ii) in water of refractive index 1.33.

Given data

- NA = 0.22
- $n_1 = 1.47$
- $n_{0(air)} = 1.00$
- $n_{0(water)} = 1.33$

Formula used

$$NA = \sqrt{(n_1^2 - n_2^2)}$$

$$\sin \theta_a = NA / n_0$$

Step-by-step solution

1. Cladding index: $n_2^2 = n_1^2 - NA^2 = (1.47)^2 - (0.22)^2$.
2. $n_2^2 = 2.1609 - 0.0484 = 2.1125$, so $n_2 = \sqrt{2.1125} = 1.4534$.
3. (i) In air: $\sin \theta_a = 0.22 / 1.00 = 0.22$, so $\theta_a = 12.71°$.
4. (ii) In water: $\sin \theta_a = 0.22 / 1.33 = 0.1654$, so $\theta_a = 9.52°$.

Final answer

$$\mathbf{n_2 = 1.453;\ \theta_a = 12.7° \text{ in air, } 9.5° \text{ in water}}$$

Exam tip

Part (ii) is the trap. NA is a property of the **fibre** and does not change; the acceptance angle depends on the **outside medium** and does change. Denser surroundings mean a narrower acceptance cone.

Unit I — Frequently Asked University Questions

How to read this section

These are the *kinds* of question that examiners have asked most often from this unit in recent SPPU Engineering Physics papers. They are written here in our own words as a revision guide — they are not reproduced from any question paper, and they are not a prediction of your paper.

Very important — expect one of these as a long question every year

⚠ COMMON MISTAKES

1. Explain the construction and working of a CO_2 laser with a neat labelled diagram and an energy-level diagram. State the function of each gas.
2. Derive the expression for the numerical aperture of a step-index optical fibre. Define acceptance angle and acceptance cone.
3. Explain the recording and reconstruction of a hologram with neat diagrams. State any four applications of holography.
4. What is population inversion? Explain why it cannot be achieved in a two-level system, and describe three- and four-level pumping schemes.

Frequently asked

IMPORTANT NOTE

1. Distinguish between spontaneous and stimulated emission (any five points).
2. Explain the characteristics of laser light. Why is laser light coherent?
3. State the conditions for total internal reflection and derive the expression for the critical angle.
4. Explain the causes of attenuation in optical fibres and sketch the attenuation-versus-wavelength curve.
5. Compare single-mode and multimode fibres, and step-index with graded-index fibres.
6. Draw the block diagram of an optical fibre communication system and state its advantages over copper cable.

Typical long-answer questions (6–8 marks)

⚙ REAL-LIFE APPLICATIONS

1. With a neat diagram, explain the three components essential to every laser and the function of each.
2. Explain holographic interferometry and its use in non-destructive testing.
3. Discuss the engineering applications of lasers in information technology, medicine and industry, with the physical reason for each.
4. Explain the classification of optical fibres and justify why graded-index fibre has a higher bandwidth than step-index fibre.

Typical short-answer questions (3–4 marks)

★ **EXAM TIP**

1. Define active medium, active centre and resonant cavity.
2. What is a metastable state and why is it essential for laser action?
3. Define numerical aperture and state its significance.
4. What is the attenuation coefficient? Give its unit.
5. State any four advantages of optical fibre communication.
6. Why is a hologram able to reproduce the whole object even when broken?
7. What is the V-number and what does it tell you?
8. Why must the optics of a CO_2 laser be made of germanium rather than glass?

Numericals commonly set

✔ **REMEMBER THESE POINTS**

1. Photon energy from a given wavelength, or the wavelength from a given level separation.
2. Population ratio from the Boltzmann relation at a stated temperature.
3. Beam divergence from two diameters, and the spot size at a distance.
4. Critical angle, NA, acceptance angle and Δ from n_1 and n_2 — the most common of all.
5. V-number and number of modes, with a comment on whether the fibre is single mode.
6. Attenuation coefficient from input and output powers, or output power from α and L.
7. Peak power and photon number for a pulsed laser.

Theory questions that ask for reasoning

◆ **DEFINITION**

1. Why can a two-level system never produce laser action?
2. Why does a laser beam diverge so little compared with a torch?
3. Why is the third window (1.55 μm) preferred for long-haul communication?
4. Why does Rayleigh scattering fall so steeply with wavelength?
5. Why is a large numerical aperture not always desirable?
6. Why must the same reference beam be used to reconstruct a hologram?

Unit I — Important Definitions

A definition question is the cheapest mark in the paper. Learn these so that you can write them exactly, then add one line of explanation or one example if the marks allow.

Term	Definition to learn
Absorption	The process in which an atom in a lower energy level takes up a photon of energy $h\nu = E_2 - E_1$ and jumps to the higher level.
Spontaneous emission	The emission of a photon by an excited atom falling to a lower level entirely on its own, without any external trigger; the photon has random phase and direction.

Term	Definition to learn
Stimulated emission	The emission of a photon by an excited atom forced down by an incident photon of the same energy; the emitted photon is identical to the incident one in frequency, phase, direction and polarisation.
Population inversion	The non-equilibrium condition in which the number of atoms in a higher energy level exceeds the number in a lower level ($N_2 > N_1$).
Pumping	The process of supplying energy from an external source to raise atoms to a higher energy level and so create population inversion.
Active medium	The material inside which population inversion is created and stimulated emission occurs.
Active centre	The particular atom, ion or molecule in the active medium that takes part in the laser transition.
Resonant cavity	The arrangement of two parallel mirrors on either side of the active medium that provides optical feedback and selects the direction and wavelength of the output.
Metastable state	An excited energy level with an unusually long lifetime ($\approx 10^{-3}$ s), in which atoms can accumulate to produce population inversion.
Threshold condition	The condition in which the gain per round trip in the cavity just equals the total loss; laser action begins above it.
Coherence	The property of a wave in which a definite phase relationship is maintained — in time (temporal) or across the wavefront (spatial).
Coherence length	The distance over which a wave maintains a predictable phase; $L_c = c\tau_c$.
Beam divergence	The angular spread of a laser beam as it propagates, $\theta = (d_2 - d_1)/(x_2 - x_1)$.
Holography	The technique of recording the amplitude and phase of a light wave as an interference pattern and later reconstructing it to give a three-dimensional image.
Hologram	The developed recording medium carrying the interference pattern of the object and reference beams.
Optical fibre	A thin flexible cylinder of glass or plastic that guides light along its length by total internal reflection.
Critical angle	The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°; $\sin\theta_c = n_2/n_1$.
Total internal reflection	The complete reflection of light back into the denser medium when it strikes the boundary with a rarer medium at an angle greater than the critical angle.
Acceptance angle	The maximum angle from the fibre axis at which light can enter the core and still be guided by total internal reflection.
Acceptance cone	The cone of half-angle θ_a at the input face of a fibre, within which entering rays are guided.
Numerical aperture	The sine of the acceptance angle; a measure of the light-gathering ability of a fibre, $NA = \sqrt{n_1^2 - n_2^2}$.
Fractional index change	$\Delta = (n_1 - n_2)/n_1$, the relative difference between core and cladding indices.
Mode	One allowed electromagnetic field pattern that can propagate along the fibre without changing its transverse shape.

Term	Definition to learn
V-number	The normalised frequency $V = (2\pi a/\lambda)\cdot NA$, which determines how many modes a fibre supports.
Attenuation	The loss of optical power suffered by a signal as it travels along a fibre.
Attenuation coefficient	The loss per unit length of fibre, $\alpha = (10/L)\log_{10}(P_{in}/P_{out})$, in dB/km.
Rayleigh scattering	Scattering of light by microscopic density and composition fluctuations frozen into the glass; it varies as $1/\lambda^4$.
Dispersion	The spreading of an optical pulse as it travels along the fibre, which limits the usable data rate.

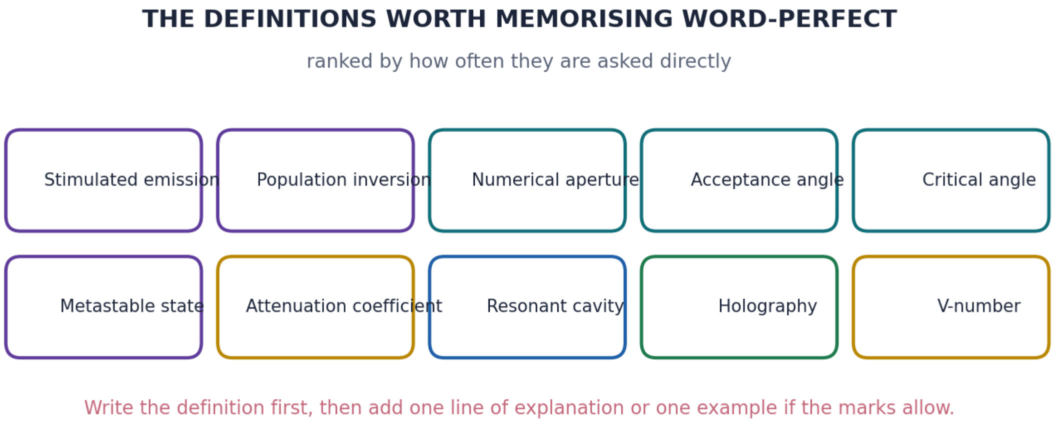


Figure 1.19 — Which definitions to learn first.

★ How to write a definition answer

A definition question is the cheapest mark in the paper, and it is also the easiest to lose. Examiners are looking for the key phrase — "identical in frequency, phase, direction and polarisation", "more atoms in the higher level than the lower", "the sine of the acceptance angle". Write the phrase, then the symbol form, then one line of explanation. Three lines, full marks.

Unit I — One-Page Revision Sheet

The last thing to read before you walk into the examination hall.

LASER — the whole story in six lines

- Pump → population inversion in the active medium.
- Metastable level lets atoms accumulate.
- One spontaneous photon triggers stimulated emission.
- Cavity mirrors feed the light back through the medium.
- Gain exceeds loss → threshold → oscillation.
- Partial mirror lets out a coherent, monochromatic, directional, intense beam.

The four characteristics

- Monochromaticity — one sharp transition + cavity mode selection.
- Coherence — temporal (phase in time) and spatial (phase across the beam).
- Directionality — only axial rays survive; $\theta \approx$ 1 mrad.
- High intensity — energy concentrated in space, time and wavelength.

CO_2 laser — the eight facts

- Four-level **molecular** gas laser; vibrational levels.
- $CO_2 : N_2 : He \approx 1 : 1 : 8$.
- N_2 (v=1) metastable, resonant with CO_2 (001).
- Upper laser level (001) asymmetric stretch.
- 10.6 µm main line; 9.6 µm secondary.
- He cools the gas and empties the lower level.
- Efficiency 20–30 % — highest of the common lasers.
- Optics of Ge or ZnSe — glass is opaque at 10.6 µm.

Holography

- Records amplitude **and** phase; photography records amplitude only.
- Recording: beam splitter → object beam + reference beam interfere on the plate.
- Reconstruction: same reference beam alone; diffraction regenerates the object wave.
- Virtual image behind, real image in front.
- Every fragment reproduces the whole object.
- Uses: security, data storage, interferometric NDT, HOEs.

Fibre — the four formulas

- $\sin \theta_c = n_2/n_1$
- $NA = \sin \theta_a = \sqrt{(n_1^2 - n_2^2)} \approx n_1\sqrt{(2\Delta)}$
- $V = (2\pi a/\lambda)\cdot NA$ — single mode when $V < 2.405$
- $\alpha = (10/L)\log_{10}(P_{in}/P_{out})$ dB/km

Attenuation — causes in three groups

- Absorption: intrinsic (UV, IR) + extrinsic (OH^-, metal ions).
- Scattering: Rayleigh ($\propto 1/\lambda^4$, dominant) + Mie.
- Bending: macrobending + microbending.
- Windows: 0.85 µm, 1.31 µm (zero dispersion), 1.55 µm (0.2 dB/km, minimum loss).

Fibre types at a glance

- Single mode: core 8–10 µm, NA ≈ 0.12, long haul, laser source.
- Multimode: core 50–100 µm, NA ≈ 0.3, short links, LED source.
- Step index: abrupt boundary, zig-zag rays, more dispersion.
- Graded index: smooth profile, curved rays, transit times equalised.

Advantages of fibre communication

- Huge bandwidth · very low loss · immune to EMI · secure · light and small · electrically isolated · no crosstalk · cheap raw material.
- Limitations: fragile, splicing needs skill, carries no power, high initial cost.

Unit I — Unit Summary

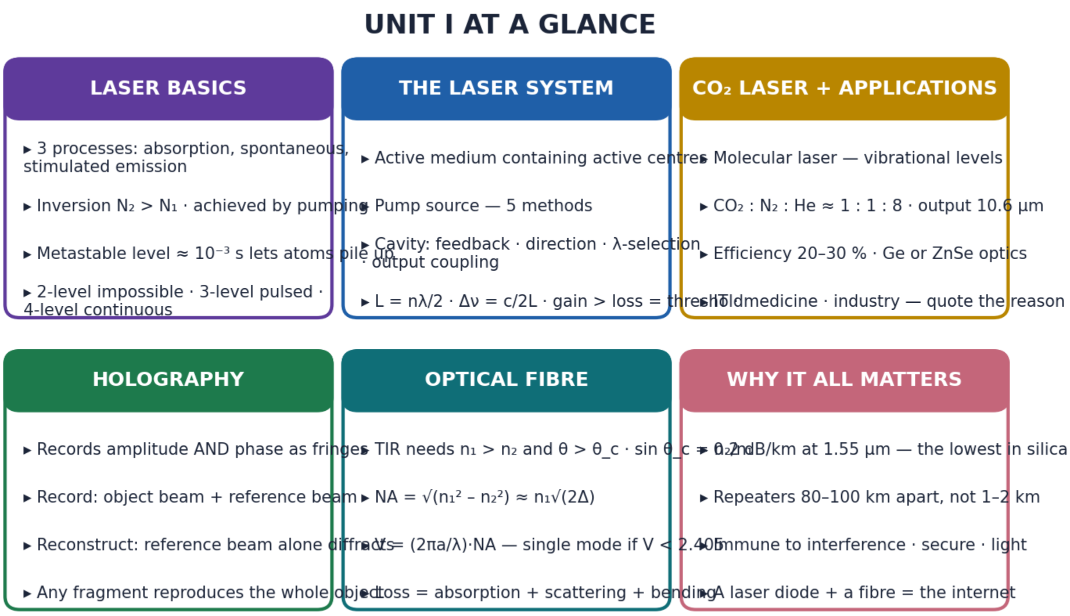


Figure 1.18 — The whole unit on one page.

What this unit was about

This unit dealt with the two great engineering achievements of modern optics: making light that is intense, pure and perfectly directed, and then carrying it long distances without losing it. Everything in the unit hangs on those two aims.

The laser half

We began with the three ways light and matter exchange energy — absorption, spontaneous emission and stimulated emission — and saw that only the third can multiply identical photons. Since thermal equilibrium always leaves the lower level crowded, stimulated emission normally loses to absorption. Breaking that equilibrium is the whole problem, and the answer is population inversion, achieved by pumping through a three- or four-level scheme with a metastable level to hold the atoms.

A laser then needs only three things: an active medium containing the active centres, a pump, and a resonant cavity. The cavity does four jobs — feedback, direction selection, wavelength selection and output coupling — and it is the cavity that gives laser light its four famous characteristics of monochromaticity, coherence, directionality and high intensity.

The CO_2 laser was studied as the representative example. Its laser levels are vibrational rather than electronic, which puts its output at 10.6 µm in the infrared. Nitrogen stores pump energy in a metastable vibrational level and passes it resonantly to CO_2; helium cools the gas and empties the lower laser level. The result is 20–30 % efficiency and the ability to deliver kilowatts continuously, which is why it dominates industrial cutting and welding.

Holography was then treated as the application that uses coherence most directly. An ordinary photograph throws away phase and is therefore flat; a hologram converts phase differences into recordable intensity fringes by interfering the object beam with a reference beam. Illuminating the developed plate with the same reference beam regenerates the object wave and produces a true three-dimensional image — with the striking property that every fragment of the plate still carries the whole object.

The optical fibre half

The second half asked how to transport that light. The answer is total internal reflection at a core–cladding boundary, which is loss-free and can be repeated indefinitely. From Snell's law we obtained the critical angle, and from the geometry at the input face we derived the single most important fibre formula, $NA = \sqrt{(n_1^2 - n_2^2)}$.

Fibres were then classified in two independent ways — by number of modes and by index profile — and we saw the engineering trade-off that runs through the whole subject: a large numerical aperture gathers more light but admits more paths, and more paths mean more dispersion and less bandwidth. Single-mode step-index fibre wins for long distances; graded-index multimode fibre is the compromise for medium-range links.

Finally we examined why signals fade. Absorption, scattering and bending each contribute, and their wavelength dependence is opposite in sign — Rayleigh scattering falls as $1/\lambda^4$ while infrared absorption rises — leaving a minimum near 1.55 µm where the loss is only about 0.2 dB/km. That single number is what makes intercontinental optical communication possible, and it explains the long list of advantages fibre holds over copper.

How the two halves join

Put the halves together and you have modern communication: a semiconductor laser, modulated at gigahertz rates, launching coherent light into a single-mode fibre with an attenuation of a fifth of a decibel per kilometre. Every idea in this unit is present in that one sentence.

> **★ If you remember only one thing from this unit**
> Stimulated emission plus a resonant cavity gives you a coherent beam; total internal reflection between a core and a cladding lets you carry it. Everything else in this unit is detail hung on those two ideas.

Physical Constants and Useful Conversions

Keep these at the front of your mind in the examination hall. Most SPPU numericals use the rounded values shown in the third column.

Constant	Symbol	Value to use	SI unit
Planck's constant	h	6.626×10^{-34}	J s
Reduced Planck constant	$\hbar = h/2\pi$	1.055×10^{-34}	J s
Speed of light in vacuum	c	3×10^{8}	$m\ s^{-1}$
Charge of an electron	e	1.6×10^{-19}	C
Rest mass of an electron	m_e	9.11×10^{-31}	kg
Rest mass of a proton	m_p	1.673×10^{-27}	kg
Boltzmann constant	k_B	1.38×10^{-23}	$J\ K^{-1}$
Boltzmann constant in eV	k_B	8.617×10^{-5}	$eV\ K^{-1}$
Permittivity of free space	ε_0	8.854×10^{-12}	$F\ m^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	$H\ m^{-1}$
Avogadro's number	N_A	6.022×10^{23}	mol^{-1}
Electronvolt	1 eV	1.6×10^{-19}	J
Angstrom	1 Å	10^{-10}	m
Nanometre	1 nm	10^{-9}	m
Micrometre	1 µm	10^{-6}	m
$k_B T$ at 300 K	—	0.026 (≈ 1/38.7)	eV

Handy shortcuts

- **E (eV) = 1240 / λ (nm)** — converts a wavelength straight to a photon energy. The fastest check in the whole subject.
- **λ (nm) = 1.226 / √V** — de Broglie wavelength of an electron accelerated through V volts (Unit II).
- A loss of **3 dB** halves the power; **10 dB** divides it by ten.
- 1 rad = 57.3°; 1 mrad spreads a beam by about 1 m in 1 km.

★ A last piece of advice

Write the constants you need at the top of your answer sheet before you start the numericals. It costs a minute and stops you hunting for them under time pressure.