Unit I — Formula Sheet

Every formula that appears in this unit, in one place. Learn the equation **and** the meaning of each symbol — SPPU examiners regularly ask for the meaning of terms as a separate part of a question.

Quantity / law	Equation	Symbols and SI units	When you use it
Photon energy	$E = h\nu = hc/\lambda$	E in J; $h = 6.626\times10^{-34}$ J s; ν in Hz; λ in m; $c = 3\times10^{8}$ m s^{-1}	Finding the energy of the laser transition, or λ from the level gap.
Boltzmann population ratio	$N_2/N_1 = e^{-\Delta E/k_BT}$	ΔE in J; $k_B = 1.38\times10^{-23}$ J K^{-1}; T in K	Showing that inversion cannot occur at equilibrium.
Einstein relation	$A_{21}/B_{21} = 8\pi h\nu^3/c^3$	A_{21} in s^{-1}; B_{21} in $m^3\ J^{-1}\ s^{-2}$	Comparing spontaneous with stimulated emission at a given frequency.
Cavity mode condition	$L = n\lambda/2,\ \ \nu_n = nc/2L$	L in m; n an integer; λ in m; ν in Hz	Finding the allowed longitudinal modes of a cavity.
Mode spacing	$\Delta\nu = c/2L$	Δν in Hz; L in m	Finding the frequency gap between adjacent modes.
Beam divergence	$\theta = (d_2 - d_1)/(x_2 - x_1)$	θ in rad; d and x in m	Numericals on how much a laser beam spreads.
Coherence length	$L_c = c\ \tau_c = \lambda^2/\Delta\lambda$	L_c in m; τ_c in s; λ, Δλ in m	Judging whether a source is coherent enough for holography.
Critical angle	$\theta_c = \sin^{-1}(n_2/n_1)$	θ_c in degree; n_1, n_2 dimensionless	Finding the minimum angle for total internal reflection.
Numerical aperture	$NA = \sin\theta_a = \sqrt{(n_1^2 - n_2^2)}$	NA and n dimensionless; θ_a in degree	The most frequently used fibre formula — light-gathering power.
NA in terms of Δ	$NA \approx n_1\sqrt{(2\Delta)},\ \Delta = (n_1 - n_2)/n_1$	Δ dimensionless (often quoted as a percentage)	When the fractional index change is given instead of n_2.
Acceptance angle	$\theta_a = \sin^{-1}(NA/n_0)$	$n_0 = 1$ for air	Converting NA to the acceptance angle; full cone = $2\theta_a$.
V-number	$V = (2\pi a/\lambda)\sqrt{(n_1^2 - n_2^2)}$	a = core **radius** in m; λ in m; V dimensionless	Deciding whether a fibre is single mode (V < 2.405).
Number of modes	$N \approx V^2/2$ (step), $N \approx V^2/4$ (graded)	N dimensionless	Estimating how many modes a fibre carries.

Quantity / law	Equation	Symbols and SI units	When you use it
Attenuation coefficient	$\alpha = (10/L)\log_{10}(P_{in}/P_{out})$	α in dB km^{-1}; L in km; P in W	Finding the loss of a fibre link, or the output power after L km.
Output power after L km	$P_{out} = P_{in} \times 10^{-\alpha L/10}$	P in W or mW; α in dB/km; L in km	The reverse calculation — how much power survives.

Unit I — Numerical Problems

Problems are graded: the first few are direct substitution, the later ones need two or three steps or a rearrangement. Work each one on paper before reading the solution.

HOW TO ATTACK ANY FIBRE NUMERICAL

1. **What is given?**
   n_1 and n_2? · NA and n_1? · Δ? · powers and length?
2. **Convert units first**
   μm → m · km stays km for α · angles in degrees
3. **Pick the formula**
   $NA = \sqrt{n_1^2 - n_2^2}$ · $V = (2\pi a/\lambda)NA$ · $\alpha = (10/L)\log(P_{in}/P_{out})$
4. **Watch the classic traps**
   radius not diameter · θ_a not $2\theta_a$ · $\log_{10}$ not ln
5. **Cross-check**
   $NA \approx n_1\sqrt{2\Delta}$ · 3 dB halves the power · V < 2.405 → single mode

Figure 1.17 — A reliable routine for any fibre numerical.

➤ Problem 1

A He–Ne laser emits light of wavelength 632.8 nm. Calculate the energy of one photon in joule and in electronvolt.

Given data

- $\lambda = 632.8$ nm $= 632.8 \times 10^{-9}$ m
- $h = 6.626 \times 10^{-34}$ J s
- $c = 3 \times 10^{8}$ m s^{-1}

Formula used

$$E = hc / \lambda$$

Step-by-step solution

1. Substitute: $E = (6.626 \times 10^{-34} \times 3 \times 10^{8}) / (632.8 \times 10^{-9})$.
2. Numerator: $6.626 \times 3 = 19.878$, so 19.878×10^{-26} J m.
3. Divide: $E = 19.878 \times 10^{-26} / 632.8 \times 10^{-9} = 3.141 \times 10^{-19}$ J.
4. Convert to eV by dividing by 1.6×10^{-19} J/eV: $E = 3.141 \times 10^{-19} / 1.6 \times 10^{-19} = 1.963$ eV.

★ **EXAM TIP**

1. Define active medium, active centre and resonant cavity.
2. What is a metastable state and why is it essential for laser action?
3. Define numerical aperture and state its significance.
4. What is the attenuation coefficient? Give its unit.
5. State any four advantages of optical fibre communication.
6. Why is a hologram able to reproduce the whole object even when broken?
7. What is the V-number and what does it tell you?
8. Why must the optics of a CO_2 laser be made of germanium rather than glass?

Numericals commonly set

✔ **REMEMBER THESE POINTS**

1. Photon energy from a given wavelength, or the wavelength from a given level separation.
2. Population ratio from the Boltzmann relation at a stated temperature.
3. Beam divergence from two diameters, and the spot size at a distance.
4. Critical angle, NA, acceptance angle and Δ from n_1 and n_2 — the most common of all.
5. V-number and number of modes, with a comment on whether the fibre is single mode.
6. Attenuation coefficient from input and output powers, or output power from α and L.
7. Peak power and photon number for a pulsed laser.

Theory questions that ask for reasoning

◆ **DEFINITION**

1. Why can a two-level system never produce laser action?
2. Why does a laser beam diverge so little compared with a torch?
3. Why is the third window (1.55 μm) preferred for long-haul communication?
4. Why does Rayleigh scattering fall so steeply with wavelength?
5. Why is a large numerical aperture not always desirable?
6. Why must the same reference beam be used to reconstruct a hologram?

Unit I — Important Definitions

A definition question is the cheapest mark in the paper. Learn these so that you can write them exactly, then add one line of explanation or one example if the marks allow.

Term	Definition to learn
Absorption	The process in which an atom in a lower energy level takes up a photon of energy $h\nu = E_2 - E_1$ and jumps to the higher level.
Spontaneous emission	The emission of a photon by an excited atom falling to a lower level entirely on its own, without any external trigger; the photon has random phase and direction.

Term	Definition to learn
Stimulated emission	The emission of a photon by an excited atom forced down by an incident photon of the same energy; the emitted photon is identical to the incident one in frequency, phase, direction and polarisation.
Population inversion	The non-equilibrium condition in which the number of atoms in a higher energy level exceeds the number in a lower level ($N_2 > N_1$).
Pumping	The process of supplying energy from an external source to raise atoms to a higher energy level and so create population inversion.
Active medium	The material inside which population inversion is created and stimulated emission occurs.
Active centre	The particular atom, ion or molecule in the active medium that takes part in the laser transition.
Resonant cavity	The arrangement of two parallel mirrors on either side of the active medium that provides optical feedback and selects the direction and wavelength of the output.
Metastable state	An excited energy level with an unusually long lifetime ($\approx 10^{-3}$ s), in which atoms can accumulate to produce population inversion.
Threshold condition	The condition in which the gain per round trip in the cavity just equals the total loss; laser action begins above it.
Coherence	The property of a wave in which a definite phase relationship is maintained — in time (temporal) or across the wavefront (spatial).
Coherence length	The distance over which a wave maintains a predictable phase; $L_c = c\tau_c$.
Beam divergence	The angular spread of a laser beam as it propagates, $\theta = (d_2 - d_1)/(x_2 - x_1)$.
Holography	The technique of recording the amplitude and phase of a light wave as an interference pattern and later reconstructing it to give a three-dimensional image.
Hologram	The developed recording medium carrying the interference pattern of the object and reference beams.
Optical fibre	A thin flexible cylinder of glass or plastic that guides light along its length by total internal reflection.
Critical angle	The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°; $\sin\theta_c = n_2/n_1$.
Total internal reflection	The complete reflection of light back into the denser medium when it strikes the boundary with a rarer medium at an angle greater than the critical angle.
Acceptance angle	The maximum angle from the fibre axis at which light can enter the core and still be guided by total internal reflection.
Acceptance cone	The cone of half-angle θ_a at the input face of a fibre, within which entering rays are guided.
Numerical aperture	The sine of the acceptance angle; a measure of the light-gathering ability of a fibre, $NA = \sqrt{n_1^2 - n_2^2}$.
Fractional index change	$\Delta = (n_1 - n_2)/n_1$, the relative difference between core and cladding indices.
Mode	One allowed electromagnetic field pattern that can propagate along the fibre without changing its transverse shape.

Term	Definition to learn
V-number	The normalised frequency $V = (2\pi a/\lambda)\cdot NA$, which determines how many modes a fibre supports.
Attenuation	The loss of optical power suffered by a signal as it travels along a fibre.
Attenuation coefficient	The loss per unit length of fibre, $\alpha = (10/L)\log_{10}(P_{in}/P_{out})$, in dB/km.
Rayleigh scattering	Scattering of light by microscopic density and composition fluctuations frozen into the glass; it varies as $1/\lambda^4$.
Dispersion	The spreading of an optical pulse as it travels along the fibre, which limits the usable data rate.

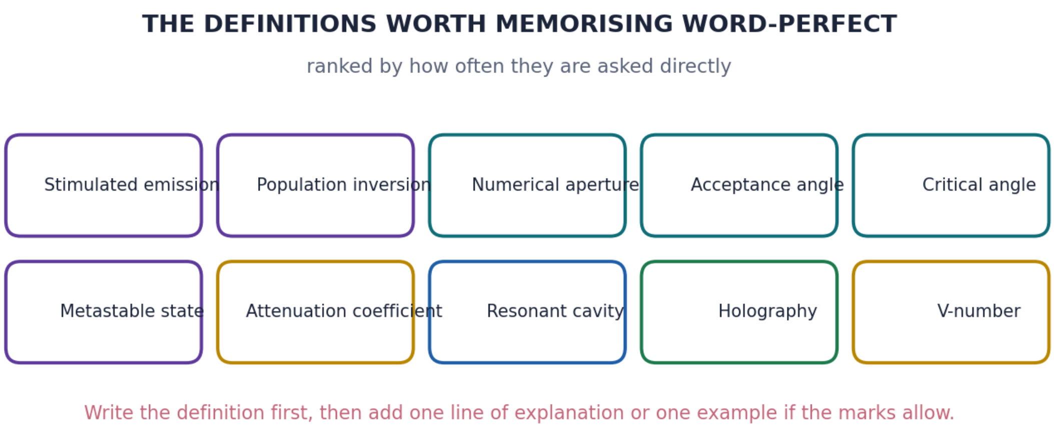

Figure 1.19 — Which definitions to learn first.

★ How to write a definition answer

A definition question is the cheapest mark in the paper, and it is also the easiest to lose. Examiners are looking for the key phrase — “identical in frequency, phase, direction and polarisation”, “more atoms in the higher level than the lower”, “the sine of the acceptance angle”. Write the phrase, then the symbol form, then one line of explanation. Three lines, full marks.